LÖSUNGSBLATT WTH. und STOCH. PROZESSE

24.6.2004

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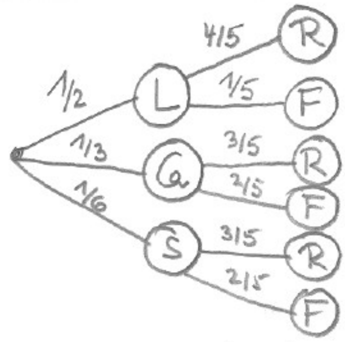
① (a)



(4P)

(b) $P(R) = P(R|L)P(L) + P(R|G)P(G) + P(R|S)P(S)$

$$= 4/5 \cdot \frac{1}{2} + \frac{3}{5} \cdot \frac{1}{3} + \frac{3}{5} \cdot \frac{1}{6} = \underline{\frac{7}{10}} \quad (6P)$$

(c) $$P(G|F) \underset{\substack{\uparrow \\ \text{S.v. Bayes}}}{=} \frac{P(F|G)\,P(G)}{1-P(R)} = \frac{\frac{2}{5}\cdot\frac{1}{3}}{\frac{3}{10}} = \underline{\underline{\frac{4}{9}}} \quad (6P)$$

(d) $n = 10$ Fragen, $p = P(R) = \frac{7}{10}$, $X = \#(\text{richtige Antworten}) \sim B(10, \frac{7}{10})$

$$P_X(X \geq 8) = \sum_{i=8}^{10} \binom{10}{i} \left(\frac{7}{10}\right)^i \left(\frac{3}{10}\right)^{10-i} = \left(\frac{7}{10}\right)^8 \left(45\left(\frac{3}{10}\right)^2 + 10 \cdot \frac{7}{10} \cdot \frac{3}{10} + \left(\frac{7}{10}\right)^2\right) = \underline{0.3828} \quad (4P)$$

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② $X \sim B(n, p = \alpha/360)$

(a) $$\varphi(p) = P_X(X=2) = \binom{10}{2} p^2 (1-p)^8 \Rightarrow \varphi'(p) = 2\binom{10}{2} p(1-p)^7 \underbrace{[(1-p) - 4p]}_{=0} \overset{!}{=} 0 \quad (8P)$$

$$\Rightarrow \underline{p = \frac{1}{5}}, \text{ d.h. } \alpha = \frac{360}{5} = \underline{72^\circ}$$

(b) $\underline{p = \frac{1}{4}}$: $P_X(X \geq 1) = 1 - P_X(X=0) \geq 0.95 \Rightarrow P_X(X=0) < 0.05$

$$\Leftrightarrow (1-p)^n < 0.05 \Leftrightarrow \left(\frac{3}{4}\right)^n < 0.05 \Leftrightarrow n \underbrace{\log\left(\frac{3}{4}\right)}_{<0} < 0.05 \Leftrightarrow n > \frac{\log 0.05}{\log 0.75} = \underline{10.4} \quad (6P)$$

$\Rightarrow \underline{n \geq 11}$ Drehungen

(c)

$G = k$	-2	1	10
$X = i$	0	1	2
$P_X(X=i)$	$9/16$	$6/16$	$1/16$

$n = 2, p = \frac{1}{4}$: $P_X(X=0) = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$, $P_X(X=1) = 2 \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{6}{16}$.

$P_X(X=2) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$E(G) = -2 \cdot \frac{9}{16} + 1 \cdot \frac{6}{16} + 10 \cdot \frac{1}{16} = \underline{-\frac{1}{8}} \Rightarrow$ Spiel ist unfair (6P)

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③ $P_X(X \leq a) = 0.25$; $P_X(X \geq b) = 0.6$.

(a) $X \sim Exp(1/4)$: $F_X(x) = 1 - e^{-x/4} \Rightarrow F_X(a) = 1 - e^{-a/4} = 0.25 \Rightarrow a = -4 \ln 0.75 \Rightarrow \underline{a = 1.1507}$

$1 - F_X(b) = e^{-b/4} = 0.6 \Rightarrow b = -4 \ln 0.6 \Rightarrow \underline{b = 2.043}$

(6P)

(b) $X \sim N(4,2)$: $F_X(x) = \Phi\left(\frac{x-4}{2}\right)$; $F_X(a) = \Phi\Big(\underbrace{\frac{a-4}{2}}_{z}\Big) = 0.25 \Rightarrow 1 - \Phi(z) \underset{\substack{\uparrow \\ \text{Eigenschaft von } \Phi}}{=} \Phi(-z) = 0.75$

$\Rightarrow$ Tab. 7.1: $-z = 0.67 \Rightarrow z = \frac{a-4}{2} = -0.67 \Rightarrow a = -2 \times 0.67 + 4 = \underline{2.66}$

$1 - F_X(b) = 1 - \Phi\left(\frac{b-4}{2}\right) = \Phi\left(-\frac{b-4}{2}\right) = 0.6 \underset{\text{Tab. 7.1}}{\Rightarrow} -\frac{b-4}{2} = 0.26 \Rightarrow b = -2 \times 0.26 + 4 = \underline{3.49}$

(8P)

(c) $X \sim U(-2,6)$: $F_X(x) = \frac{x+2}{8}$; $F_X(a) = \frac{a+2}{8} \overset{!}{=} 0.25 \Rightarrow a = 8 \cdot 0.25 - 2 = \underline{\underline{0}}$

$1 - F_X(b) = 1 - \frac{b+2}{8} = 0.6 \Rightarrow \frac{b+2}{8} = 0.4 \Rightarrow b = 8 \cdot 0.4 - 2 = \underline{1.2}$ (6P)

4

(a) $F_X(x) = \int_0^x a^2 t e^{-at}\,dt = -ate^{-at}\Big]_0^x + \underbrace{\int_0^x a e^{-at}\,dt}_{Exp(a)}$

$u=t,\ v' = a^2 e^{-at}$

$u'=1,\ v = -ae^{-at}$

$= -axe^{-ax} + 1 - e^{-ax} = \underline{1 - e^{-ax}(1+ax)},\ x>0$ (8P)

(b) $P_X(800 < X < 1100) = F_X(1100) - F_X(800) = e^{-2/5}(1+2/5) - e^{-11/20}(1+\frac{11}{20})$

$a = 1/2000$

$= 0.93845 - 0.89427 = \underline{0.044}$ (6P)

(c) $P_X(X > 2100) = 1 - F_X(2100) = e^{-21/20}(1+\frac{21}{20}) = 0.7174$: größer

$P_X(X \le 1500) = F_X(1500) = 1 - e^{-3/4}(1+\frac{3}{4}) = 1 - 0.8266 = \underline{0.1734}$ (6P)

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5 $N_t \sim P(\lambda t) = P(t/20)$. $P(N_t = k) = \frac{(t/20)^k}{k!} e^{-t/20}$, $k = 0,1,2,\ldots$

(a) $P(N_6 \ge 1) = 1 - P(N_6 = 0) = 1 - e^{-6/20} = 1 - e^{-0.3} = \underline{0.2592}$ (4P)

(b) $P(N_8 - N_2 = 1, N_{12} - N_8 = 0) = P(N_6 = 1, N_4 = 0) \overset{\text{unabh.}}{=} P(N_6 = 1)\,P(N_4 = 0)$

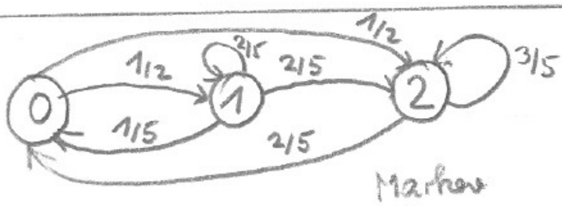
$= 0.3e^{-0.3} \cdot e^{-0.2} = \underline{0.18196}$ (6P)

(c) $P(N_8 - N_4 = 1 \mid N_4 - N_0 = 0) \overset{\text{unabh.}}{=} P(N_8 - N_4 = 1) = P(N_4 = 1) = 0.2e^{-0.2} = \underline{0.16375}$ (4P)

(d) $P(T_3 \ge 10) \Leftrightarrow P(N_{10} \le 2) = e^{-1/2}(1 + \frac{1}{2} + \frac{1}{8}) = \underline{0.985}$ (6P)

6

(a)



(4P)

(b) $P(X_2 = 0 \mid X_1 = 2, X_0 = 1) \overset{\text{Markov Eig.}}{=} P(X_2 = 0 \mid X_1 = 2) = p_{20} = \underline{\underline{\frac{2}{5}}}$

$P(X_2 = 0, X_1 = 2 \mid X_0 = 1) = P(X_2 = 0 \mid X_1 = 2, X_0 = 1)\,P(X_1 = 2 \mid X_0 = 1)$

$\overset{\text{ME}}{=} P(X_2 = 0 \mid X_1 = 2)\,P(X_1 = 2 \mid X_0 = 1)$

$= p_{20} \cdot p_{12} = \frac{2}{5} \cdot \frac{2}{5} = \underline{\frac{4}{25}}$ (6P)

(c) $P(X_{n+1} = 2, X_n = 1 \mid X_{n-1} = 0) \overset{(b)}{=} P(X_{n+1} = 2 \mid X_n = 1)\,P(X_n = 1 \mid X_{n-1} = 0)$

$= p_{12} \cdot p_{01} = \frac{2}{5} \cdot \frac{1}{2} = \underline{\underline{\frac{1}{5}}}$ (4P)

(d) $p^{(0)} = (\frac{1}{5}, \frac{2}{5}, \frac{2}{5})$, $p^{(1)} = p^{(0)} \cdot \underset{\sim}{P} = (\frac{1}{5}\ \frac{2}{5}\ \frac{2}{5}) \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/5 & 2/5 & 2/5 \\ 2/5 & 0 & 3/5 \end{pmatrix} = (\frac{12}{50}, \frac{13}{50}, \frac{25}{50})$

$P(X_1 = 1) = p_1^{(1)} = \underline{\frac{13}{50}}$

$P(X_1 = 1, X_2 = 0) \underset{\text{bed.W!}}{=} P(X_2 = 0 \mid X_1 = 1)\,P(X_1 = 1) = p_{10} \cdot p_1^{(1)} = \frac{1}{5} \cdot \frac{13}{50} = \underline{\frac{13}{250}}$ (6P)