A bivariate compound Poisson frailty model

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A new frailty model is suggested for analysis of bivariate time-to-event data. The model is an extension of the correlated PVF frailty model (correlated three-parameter frailty model) introduced by Yashin and Iachine (1999) and has never before been considered in bivariate survival studies. It is based on a bivariate extension of the compound Poisson frailty model introduced to univariate survival analysis by Aalen (1988, 1992). It is also related to the extended correlated gamma frailty model by Wienke et al. (2003). It allows for a non-susceptible fraction in the population, overcoming the common assumption in survival analysis that all individuals are susceptible to the event under study. The unconditional survival function of the correlated compound Poisson frailty model is given by:

$$
\begin{aligned}
S(t_1,t_2) &= S(t_1)^{1-\rho} S(t_2)^{1-\rho} \\
&\quad * \exp\{\frac{\rho(1-\gamma)}{\gamma\sigma^2}\big(1-((1-\frac{\gamma\sigma^2}{1-\gamma}\ln(S(t_1)))^{1/\gamma}+(1-\frac{\gamma\sigma^2}{1-\gamma}\ln(S(t_2)))^{1/\gamma}-1)\big)^{\gamma}\}.
\end{aligned}
$$

Parameter γ divides the class of distributions in two major subfamilies: For $\gamma \geq 0$, the distribution is a power variance function distribution (PVF). The extension to $\gamma < 0$ in the univariate case was suggested by Aalen (1988) and shown to yield the compound Poisson distribution. The two subclasses are separated by the gamma distribution ($\gamma = 0$). The inverse Gaussian model is given by $\gamma = 0.5$. Parameter values of $\gamma < 0$ imply the existence of a non-susceptible fraction in the population. The model is applied to breast cancer incidence data of 5857 Swedish twin pairs (born between 1886 - 1925) to estimate the size of the susceptible fraction. Results are compared to the analysis by Wienke et al. (2003).

References:

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