# *Modeling Negative Rates*

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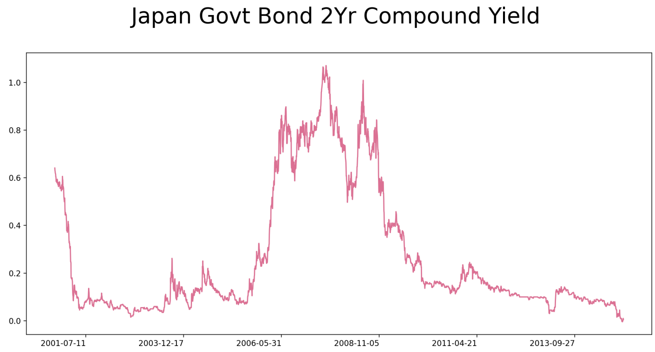
# Interest rates




Japan government bond 2 year compound yield from 2000.10.02 to 2014.12.17

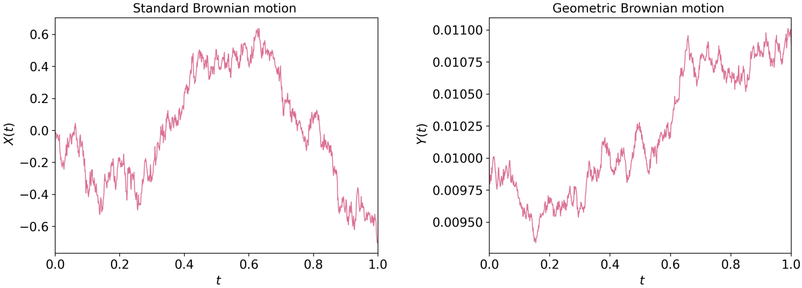
# Purpose

- goal: modeling yield curves
- trajectories similar to what we have seen in the past
- trajectories that contain future information (implied statistical features which can be observed in current prices)
- purpose: pricing, risk managament

## *Steps of the analysis*

- 4 main steps in the analysis
  1. finding the right trajectories, model class
  2. calibration
  3. pricing financial assets
  4. risk management (hedging)
- the focus is on the first point $\rightarrow$ **finding statistically appropriate trajectories**

# Brownian motion




- Brownian motion → normal distribution
- geometric Brownian motion → lognormal distribution
- drive processes in the models

# *SABR model*

- standard practice in the financial world
- stochastic volatility model *[Hagan (2002)]*
- captures the volatility smile

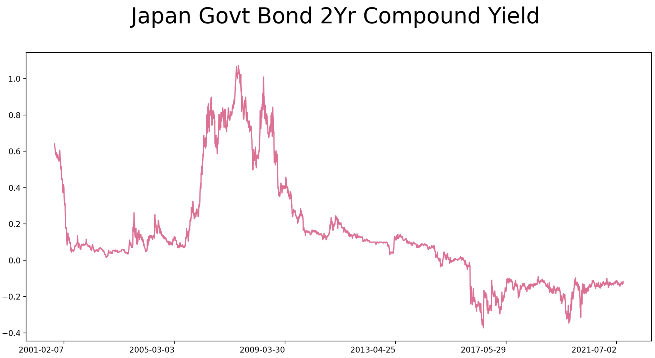
$$df(t) = f^{\beta}(t)\nu(t)dW(t)$$
$$d\nu(t) = \alpha\nu(t)dZ(t)$$

- where $\nu(0), \alpha \in \mathbb{R}^{+}$ and $0 \leq \beta \leq 1$
- $W(t)$ and $Z(t)$ are one dimensional Wiener processes and $d[W,Z](t) = \rho dt$, where $\rho \in [-1,1]$.

# SABR model

- four parameter set: $(\alpha, \nu, \beta, \rho)$
- $\alpha = \nu(0)$: initial volatility
- $\nu(s)$: the volatility of the volatility
- $\rho$: correlation between the two Wiener process $\rightarrow$ also the correlation between the rate and the volatility, the so-called skew
- $\beta$: CEV (Constant Elasticity Variance) parameter (the power of the forward rates)

# Negative rates




Japan government bond 2 year compound yield from 2000.10.02 to 2021.10.01

# Phenomean of negative rates

- due to economical conditions central banks were forced to lower interest rates
- negative interest rates appeared
- new phenomean in the markets $\rightarrow$ uncertainty
- models for pricing financial assets haven't worked anymore
- mathematical models have to be extended

# Desirable features

- **heavy-tailed distribution:** the probability of extreme values is higher than in the case of normal distribution
- **left oblique-right extending distribution:** lower interest rates are more common than high ones
- appearance of **negative values**

# Free Boundary SABR model

- taking the absolute value of the forward rate
- natural extension of the SABR model

$$df(t) = |f(t)|^{\beta}\nu(t)dW(t)$$

$$d\nu(t) = h(t)\nu(t)dZ(t)$$

- $W(t)$ and $Z(t)$ are correlating Wiener-processes, where $d[W, Z](t) = \rho dt$
- the power $0 \leq \beta < 0,5$.

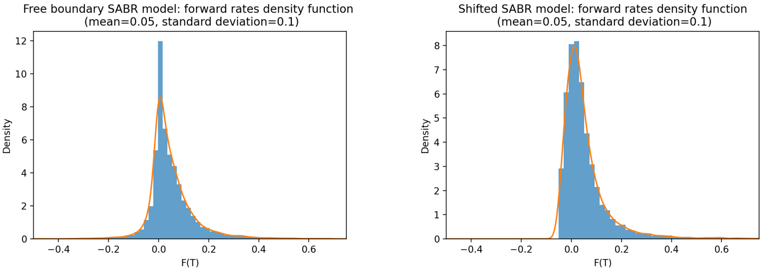
# Shifted SABR model

- add a displacement parameter to the SABR model
- $s > 0$: the shift size chosen apriori

$$df(t) = (f(t) + s)^{\beta}\nu(t)dW(t)$$
$$d\nu(t) = h(t)\nu(t)dZ(t)$$

- $0 \leq \beta \leq 1$
- $W(t)$ and $Z(t)$ are two correlating one-dimensional Wiener processes, $d[W, Z](t) = \rho dt$

# Simulations




Histograms of the free boundary SABR model and of the shifted SABR-LMM when the expected value is 5% and the standard deviation is 10%.

# Other extensions

- **Mixed model:** mix between the Gaussian affine model and the Black model
- **Heath-Jarrow-Morton framework:** captures the dynamics of the yield curve

# Important features in the practice

- how fast the model runs
- how flexible and accurate the model is
- how easy to interpret the parameters
- how well it can be calibrated

# Conclusion

- both models have advantages and disadvantages
- the models meet the expectations of the statistical features
- the next step would be the calibration of the parameters
- after that they can be used to price financial assets

# References


- *A. Antonov – M. Konikov – M. Spector* [2015]: The Free Boundary SABR: Natural Extension to Negative Rates (*Numerix*, January)
- *P. S. Hagan – A. S. Lesniewski* [2006]: LIBOR Market Model with SABR style stochastic volatility (*Wilmott Magazine*, May)
- *S. E. Shreve* [2004]: Term Structure Models. Appeared: S. E. Shreeve (Eds): *Stochastic Calculus for Finance II.: Continuous-Time Models.* New York: Springer Science + Business Media 403-459. p.
- *Y. Ueno* [2017]: Term Structure Models with Negative Interest Rates (*IMES Discussion Paper Series*, 17-E-01)
- *J. Xiong – G. Deng – X. Wang* [2020]: Extension of SABR Libor Market Model to handle negative interest rates (*Quantitative Finance and Economics*, March)

Thank you for your attention!

Any questions?