Abstract

The modeling of the negative rates

Dalma Tóth-Lakits

The economic world today is rather different from what we have seen before in history. Negative nominal interest rates for long-term maturities spread over large parts of the world. The well-proven mathematical models either do not perform well or do not perform at all in the negative range.

Therefore our aim was to collect, examine and compare the newly proposed models and to test whether they have the desirable features. Among other things, that the distributions of the rates are oblique to the left, extending to the right (the small values are more common), fat-tailed (the probability of the extreme events is higher than in a normal distribution) and also to take values below zero. These features are extremely important to keep our model consistent with the stylized facts observed in the market.

The presentation would cover the extensions of the most popular models used in the industry, for example the extensions of the SABR model, the SABR-LMM, and a mix between the Gaussian affine and the Black model. We can observe that there are three main ways to create negative term structure models. We can get models by adding a shift parameter to the model, so the value set is lowered below zero with the parameter. Obviously these models called the shifted models. They retain the distribution of the most common models, but the disadvantage is that we always have to specify the shift parameter apriori and as soon as our interest rate falls below this value the whole model has to be recalibrated. The other way is to take the absolute value of the differences of the interest rates. These models are called the free-boundary models. Obviously these models do not have a lower bound, but the distribution of the interest rates are rather different. The last way to create a new model is by mixing the favorable features of two well-known models. The Mixed model was obtained by mixing the Black and the Gaussian affine model. The model is driven by several Wiener processes which makes the simulations slower and the understanding of the parameters harder.

Finally we would present the distributions generated by the simulations written for the different models. We could conclude that each model has advantages and disadvantages, but all of them fulfills the basic requirements of an interest rate model. As a result, they can be used to price financial assets.