Computing the sampling time for induction machine parameter estimation using complex exponential series estimation

Young statistitians meeting 2021, Vorau

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Vorau, 16.10.2021.

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Introductory definitions

Dynamical systems model - state space equations:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t) &= \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)|\boldsymbol{\theta}) \\ \mathbf{y}(t) &= \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t)|\boldsymbol{\theta})\end{aligned}$$

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- $\mathbf{x}(t) \in \mathbb{R}^n$ - state variable (a function of time actually)
- $\mathbf{u}(t) \in \mathbb{R}^m$ - input variable (also a function of time)
- $\mathbf{y}(t) \in \mathbb{R}^p$ - output variable (also a function of time)
- n - model order

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- $\mathbf{f} : \mathbb{R}^{n \times m} \to \mathbb{R}^n$
- $\mathbf{g} : \mathbb{R}^{n \times m} \to \mathbb{R}^p$
- $\boldsymbol{\theta} \in \mathbb{S} \subset \mathbb{R}^r$ - parameters of the functions $\mathbf{f}$ and $\mathbf{g}$
- r - the number of parameters

Example model

Induction motor model:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{s,dq} = \mathbf{u}_{s,dq} - R_s\mathbf{i}_{s,dq} - \omega_k\mathbf{J}_r\boldsymbol{\psi}_{s,dq}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\psi}_{r,dq} = R_r\mathbf{i}_{r,dq} - (\omega_k - \omega)\mathbf{J}_r\boldsymbol{\psi}_{r,dq}$$
$$\frac{J}{p}\frac{\mathrm{d}}{\mathrm{d}t}\omega = \frac{3}{2}p\boldsymbol{\psi}_{s,dq}^T\mathbf{J}_r\mathbf{i}_{s,dq}$$

$$\begin{bmatrix}\mathbf{i}_{s,dq}\\ \mathbf{i}_{r,dq}\end{bmatrix} = \begin{bmatrix}L_s & 0 & L_m & 0\\ 0 & L_s & 0 & L_m\\ L_m & 0 & L_r & 0\\ 0 & L_m & 0 & L_r\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{\psi}_{s,dq}\\ \boldsymbol{\psi}_{r,dq}\end{bmatrix}$$

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\end{aligned}
$$

- $\theta = \begin{bmatrix} R_r & L_s & L_r & L_m \end{bmatrix}$ - model parameters
- R_s, J, J_r, ω_k, p - known constants

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- $\mathbf{u}_{s,dq}$ - input (voltage to the machine)
- $\mathbf{x} = \begin{bmatrix}\boldsymbol{\psi}_{s,dq} & \boldsymbol{\psi}_{r,dq} & \omega\end{bmatrix}^T$ - state variables
- $\mathbf{y} = \begin{bmatrix}\mathbf{i}_{s,dq} & \omega\end{bmatrix}^T$ - output (measurable) variables

Parameter estimation problem

State space model:

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Parameter estimation problem

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- $\mathbf{y}_{obs}[k]$ - sampled measurements at sampling rate T_s
- Usual sampling times for electric machinery measurements: $T_s = 10^{-4}$ s
- Usual experiment duration - around 2 s
- Costly and demanding measurements and computation

Selecting the sampling time

Sampling time criteria - linear systems

For linear systems:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

The i-th eigenvalue of $\mathbf{A}$ is λ_i - complex number.

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$$z_i = e^{\lambda_i T_s}$$

Sampling time criteria

N. K. Sinha and S. Puthenpura. Choice of the sampling interval for the identification of continuous-time systems from samples of input/output data. IEE Proceedings D - Control Theory and Applications, 132:263, 1985

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1. Measure a transient of a system with some $T_{s,meas}$
2. Somehow compute the eigenvalues of the system z_i
3. Apply the criteria to compute the sampling time

Sampling time criteria

The criteria to choose sampling time for systems:

Compute K by solving:

$$\frac{\left[|z_{min}|^{4K} + 2|z_{min}|^{2K}\cos(K\phi_z) + 1\right]^{1/2}}{|z_{min}|^{k2} - 2|z_{min}|^{K}\cos(K\phi_z) + 1} = R$$

Criteria is formed from Tustin billinear transformation accuracy

$$|\lambda_{max}|T_s \leq 0.5$$

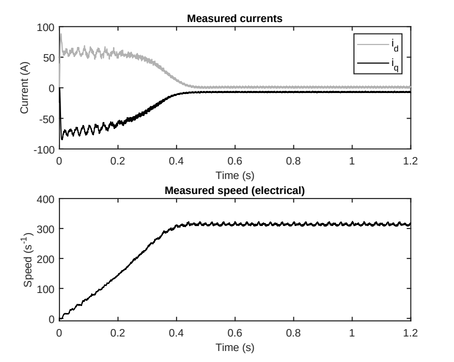
$$T_s = KT_{s,meas}$$

1. Measurments

$$\mathbf{y} = \begin{bmatrix} y_1 & \dots y_i & \dots y_p \end{bmatrix}^T$$

Perform a transient measurement as fast as possible single y_i

For induction machine measured current i_d



2. Compute the eigenvalues - somehow

Prony series of complex exponentials defined as:

$$\hat{y}[k] = \sum_{i=1}^{n} R_i \exp(j\phi_i + \lambda_i kTs) = \sum_{i=1}^{n} R_i e^{j\phi_i} z_i^k$$

Actual single input, multiple output linear system response to Dirac impulse $u(t) = \delta(0)$

$$y_i(t) = \sum_{j=1}^{n} G_{ij} \exp(\lambda_j t)$$

$$y_i[k] = \sum_{j=1}^{n} G_{ij} \exp(\lambda_j k T_s) = \sum_{j=1}^{n} G_{ij} z_j^k$$

2. Compute the eigenvalues - somehow

- Condition on the experiment - impulse response measurement
- Actual measurement - Impulse input is not possible, step input realized

Estimating the complex exponential series coefficients

Two well established methods:

1. Prony method
2. Matrix pencil method - Hua and Sarkar

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Y. Hua and T.K. Sarkar. Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise. IEEE Transactions on Acoustics, Speech, and Signal Processing, 38(5):814–824, May 1990

Matrix pencil method

Solves the nonlinear least squares problem of fitting complex exponential series to the data

The begining is forming a Henkel matrix:

$$\mathbf{H} = \begin{bmatrix} y_i[1] & \dots & y_i[n] & y_i[n+1] \\ y_i[2] & \dots & y_i[n+1] & y_i[n+2] \\ \vdots & \vdots & \vdots & \vdots \\ y_i[q-n] & \dots & y_i[q-1] & y_i[q] \end{bmatrix}$$

Matrix pencil method

The problem eventually reduces to solving generalized eigenvalue problem for z:

$$(\mathbf{H}_l + z\mathbf{H}_r)\mathbf{v} = 0 \tag{1}$$

Where:

$$\mathbf{H} = \left[\mathbf{h}_1 \ldots \mathbf{h}_{n+1}\right]$$
$$\mathbf{H}_l = \left[\mathbf{h}_1 \ldots \mathbf{h}_n\right]$$
$$\mathbf{H}_r = \left[\mathbf{h}_2 \ldots \mathbf{h}_{n+1}\right]$$

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With known z computing $G_{ij} = R_i e^{j\phi_i}$ is linear regression problem

$$y_i[k] = \sum_{j=1}^{n} G_{ij} z_j^k$$

Problems

1. The measured transient is not **impulse response**
2. The system model is **not linear**

Optimization procedure for nonlinear systems

- Nonlinear models don't have eigenvalues - Linearize the model
- At least measure the step response of the system (usually very possible)

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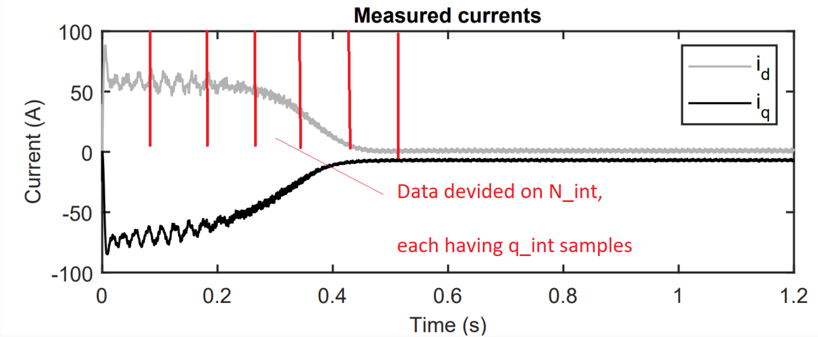
How do we select how wide is the linearization region - no prior knowledge of the system?

Step response and impulse response are initially same - how long?


Measured currents
100
50
0
-50
-100
Current (A)
0
0.2
0.4
0.6
0.8
1
1.2
Time (s)
i_d
i_q
Data devided on N_int,
each having q_int samples

Optimization procedure for nonlinear systems

Formalize the presented questions as an optimization problem

$$J_{est} = \sum_{w=1}^{N_{int}} MSE_w$$

$$MSE_w = \frac{1}{q_d} \sum_{r=(w-1)q_d}^{wq_d} (y_d[r] - \hat{y}_d[r])^2$$

$$q_d = \frac{q_{int}}{d}$$

$$N_{int} = \text{round}(\frac{q}{q_{int}})$$

y_d - decimated data $y_d = y_i(dpT_s)$

q_{int} - number of data points in each interval

N_{int} - Number of intervals

Optimization procedure for nonlinear systems

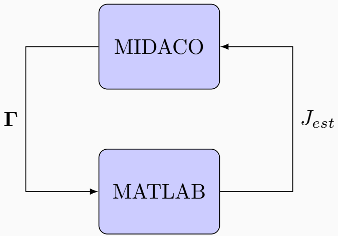
$$J_{est}(\mathbf{\Gamma}) \rightarrow \min$$

$$\hat{\mathbf{\Gamma}} = \underset{\mathbf{\Gamma} \in \mathbb{S} \subset \mathbb{N}^2}{\operatorname{argmin}} J_{est}(\mathbf{\Gamma})$$

Limited by:

$$\mathbf{\Gamma}_{LB} \leq \mathbf{\Gamma} \leq \mathbf{\Gamma}_{UB}$$

$$\mathbf{\Gamma} = \begin{bmatrix} d & q_{int} \end{bmatrix}$$



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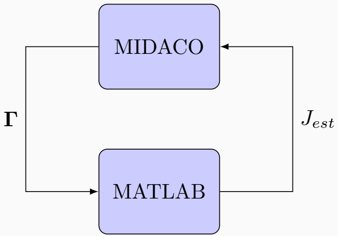

Result:

Optimal Prony approximation $\hat{y}_d$ with optimal data segmentation defined by d i q_{int}.

Result - Optimal complex exponential series

Test case - simulated data (same setting as the measurement) - Known discretized system eigenvalues

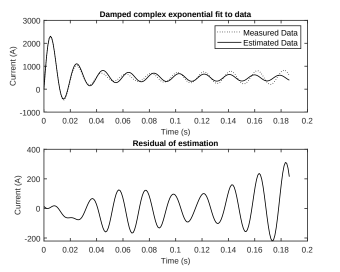

Figure 1: Simulated data

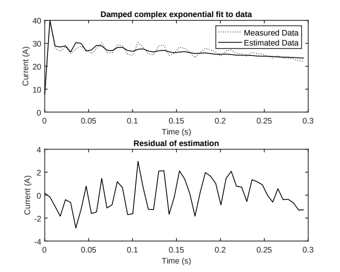

Figure 2: Actuall measurement

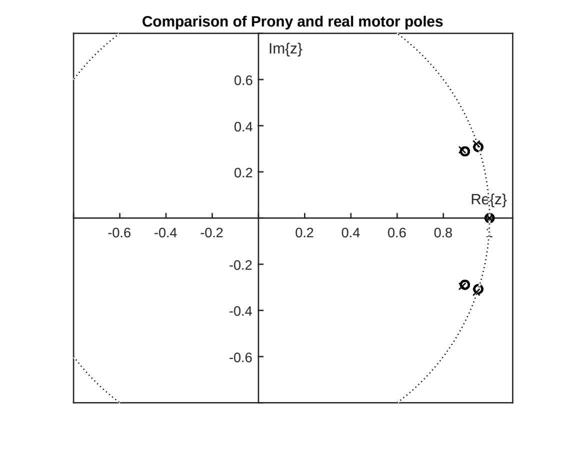

Figure 3: Prony eig. (x) vs. actual induction motor linearized system eig. (o) - sim. data

Finally!

Compute the sampling time from the Sinha criteria:

$$\frac{\left[|z_{min}|^{4K} + 2|z_{min}|^{2K}\cos(K\phi_z) + 1\right]^{1/2}}{|z_{min}|^{k2} - 2|z_{min}|^{K}\cos(K\phi_z) + 1} = R$$

R - the accuracy coefficient for the Tustin bilinear criteria :

The selected sampling times

Simulation:

R	T_s [ms]	F_s [Hz]
5	1.7746	563.52
7	1.5277	654.59
8	1.4375	695.64

Experimental:

$$R = 7$$

$$T_s = 1.187\,\mathrm{ms}$$

$$F_s = 842.61\ \mathrm{Hz}$$

Usually in practice:

$$T_s = 0.1\,\mathrm{ms}$$

$$F_s = 10000\ \mathrm{Hz}$$

Is it worth it?

Procedure is:

- Measure the data
- Compute the sampling time with:
  - Optimization procedure
  - Each iteration and each population unit estimates the complex exponential series

The parameter estimation problem - revisited

State space model:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x} = \mathbf{f}(\mathbf{x}, \mathbf{u}|\boldsymbol{\theta})$$
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Parameter estimation problem revisited

Solve

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By MIDACO optimization algorithm

$\mathbf{y}(kT_s|\boldsymbol{\theta})$ - computed with Runge-Kutta solver in Matlab

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Filip Halak, Tin Bensic, and Marinko Barukcic. Induction motor variable inductance parameter identification.In 2017 International Conference on Smart Systems and Technologies (SST). IEEE, October 2017.

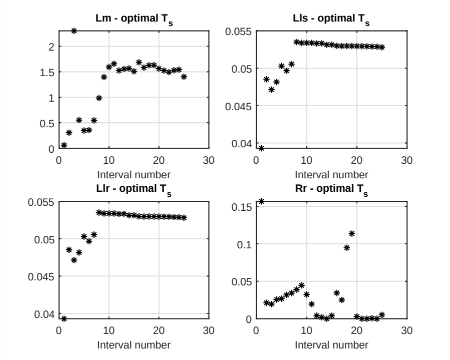

Figure 4: Parameter estimates using computed T_s, **computation time - 2 min**

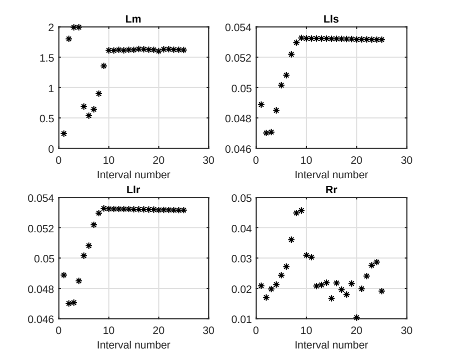

Figure 5: Parameter estimates using $T_s = 0.1$ ms, **computation time > 3 h**

Residuals of the estimation

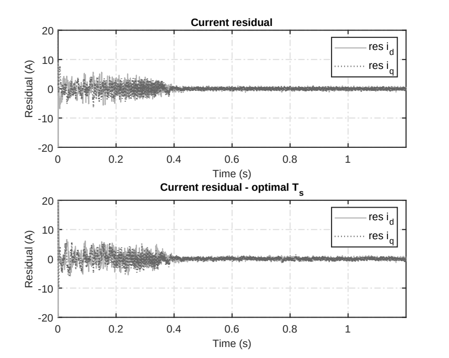

Figure 6: Current residuals

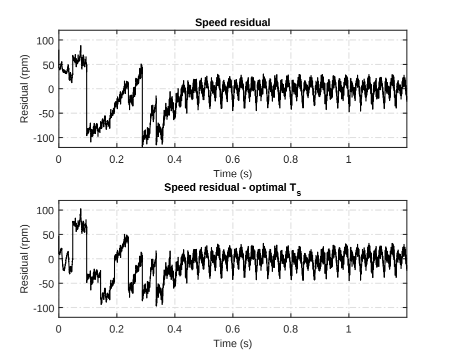

Figure 7: Speed residuals

Computational complexity of the procedure

Original T_s	Computer	Time to compute the new T_s
0.001 ms	AMD Ryzen 5 32 GB RAM	3 min
0.001 ms	Intel i7 1.gen 8 GB RAM	7 min
0.1 ms	AMD Ryzen 5 32 GB RAM	5 s
0.1 ms	Intel i7 1.gen 8 GB RAM	30 s

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Tin Benšić, Toni Varga, Marinko Barukčić, and Vedrana Jerković Štil. **Optimization procedure for computing sampling time for induction machine parameter estimation**. *Applied Sciences, 10(9):3222, May 2020*