Zeros of a two-parameter random walk

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This is joint work with D. Khoshnevisan. Let

$$\{X(i,j),\ i=1,2,\dots,j=1,2,\dots\}$$

be an array of i.i.d. r.v.'s with

$$\mathbf{P}\{X(i,j)=1\}=\mathbf{P}\{X(i,j)=-1\}=1/2$$

and put

$$S(m,n)=\sum_{i=1}^{m}\sum_{j=1}^{n}X(i,j)$$

$$I(m,n)=\begin{cases}1 & \text{if } S(m,n)=0\\ 0 & \text{otherwise,}\end{cases}$$

$$\gamma(A)=\sum_{(m,n)\in A}I(m,n)$$

where $A\subset\mathbb{Z}^2$. Our first aim is to study the number of zeros

$$\gamma_N=\gamma([0,N]^2)$$

of $S(\cdot,\cdot)$ in $[0,N]^2$.

Theorem 1. *For any $\varepsilon>0$*

$$N^{1-\varepsilon}\le\gamma_N\le N^{1+\varepsilon}\qquad\text{a.s.}$$

if N is large enough.

We are also interested in the location of the zeros on the diagonal of $[0,n]\times[0,n]$. Let

$$N_n=\sum_{1\le i\le n}\mathbf{1}_{\{S(2i,2i)=0\}}.$$

Theorem 2.

$$\lim_{n\to\infty}\frac{N_n}{\log n}=\left(\frac{2}{\pi}\right)^{1/2}\qquad\text{a.s.}$$