# A directed polymer approach to the once-oriented last passage site percolation time constant in high dimensions

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Let $\eta$ be a real random variable whose logarithmic moment generating function $\lambda(\beta) := \ln(\mathbf{E}\exp(\beta\eta))$ exists for all $\beta > 0$. Let $\nu_d$ denote the point to line last passage time constant of a once-oriented first passage site percolation in $d+1$ dimensions. Here once-oriented refers to the condition that it is always the first coordinate of a path that is increased among oriented paths along sites in $\mathbb{Z}^{(d+1)}$. One can relate $\nu_d$ to the free energy of a directed polymer model in the random field of i.i.d. copies of $\eta$ at low temperature (inverse temperature $\beta$ near infinity). Here we define the partition function of the directed polymer in the random environment $\{\eta(k,y)\}, k \geq 0, y \in \mathbb{Z}^d$, by $Z_n(\beta) = \mathbb{E}\exp(\beta\sum_{k=1}^n \eta(k,S_k))$ for a random walk $\{S_k\}$ in $\mathbb{Z}^d$, where $\mathbb{E}$ denotes the expectation relative to this random walk. The free energy is then given by $f(\beta) := \lim_{n\to\infty} \ln Z_n(\beta)/n$. The connection between $\nu_d$ and $f(\beta)$, namely $\lim_{\beta\to\infty} f(\beta)/\beta = \nu_d$, was already recognized by Comets and Yoshida (2006). Here we emphasize the fact that for a class of distributions with upper tail $\mathbf{P}(\eta > x) = \exp(-xv(x) + \delta(x))$, such that: $v(x) \nearrow \infty$ and $\delta(x) = O(x)$ as $x \to \infty$, $u(x) := xv(x)$ is strictly convex for large $x$, $\liminf_{x\to\infty} xv'(x) > 0$, and $v$ satisfies a regularity condition that makes $\exp(-u(x))$ convex for large $x$, we may obtain an asymptotic evaluation of $\nu_d$ as $d \to \infty$. These conditions admit the Poisson case where $v(x) = \ln(x) - 1$ and $\delta(x) = O(\ln(x))$. The proof involves a simple linear estimate on the free energy of the directed polymer model, namely, $f(\beta) \geq \nu_d\beta - \ln(2d)$, that is valid for all $d \geq 1$. Our condition on the upper tail of $\eta$ is reminiscent but more detailed than a similar condition given by Ben-Ari (2007). We show that $\nu_d \sim \mathbf{E}\max(\eta_1,\ldots,\eta_{2d}) \sim U(\ln(2d))$, where $U$ is the inverse of $u$. In the Poisson and Gaussian cases we obtain $\nu_d \sim \ln(2d)/\ln\ln(2d)$ and $\nu_d \sim \sqrt{2\ln(2d)}$, respectively, as $d \to \infty$. It follows in particular that if a given distribution of the above class is mixed with a distribution with a smaller tail in an appropriate sense then the last passage time constant is ruled asymptotically as $d \to \infty$ by the distribution with the larger tail.

## References

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[2] F. Comets and N. Yoshida (2006), Directed polymers in random environment are diffusive at weak disorder, *Ann. Probab.* **34** No. 5, 1746-1770. arXiv:math. PR/0411223 v2.