On the number of cutpoints of the transient NN random walk on the line

Antónia Földes

City University of New York

This is joint work with Endre Csáki and Pál Révész. Consider a nearest neighbor (NN) random walk on the line as follows: let $X_0 = 0,\ X_1, X_2, \ldots$ be a Markov chain with

$$\mathbf{P}(X_{n+1} = i+1 \mid X_n = i) = 1 - \mathbf{P}(X_{n+1} = i-1 \mid X_n = i) = \begin{cases} 1 & \text{if} \quad i = 0 \\ 1/2 + p_i & \text{if} \quad i = 1, 2, \ldots, \end{cases}$$

where $-1/2 < p_i < 1/2,\ i = 1, 2, \ldots$.

We are interested in this walk in the transient case. (A well-known result of Chung gives a criteria of transience in terms of the $\{p_i\}$ sequence.)

When $p_i \geq 0, \quad i = 1, 2, \ldots$, the sequence $\{X_i\}$ describes the motion of a particle which starts at zero, moves over the nonnegative integers and going away from 0 with a larger probability than to the direction of 0.

Call the site R a *cutpoint* if for some k, we have $X_k = R$ and $\{X_0, X_1 \ldots X_k\}$ is disjoint from $\{X_{k+1}, X_{k+2} \ldots\}$, i.e. $X_i \leq R$, $i = 0, 1, \ldots, k$, $X_k = R$ and $X_i > R$, $i = k+1, k+2, \ldots$

Call the site R a *strong cutpoint* if for some k, we have $X_k = R$, $X_i < R$, $i = 0, 1, \ldots, k-1$ and $X_i > R$, $i = k+1, k+2, \ldots$. Observe that R is a strong cutpoint if and only if the number of visits at R is exactly 1. Clearly every strong cutpoint is a cutpoint.

We will present a criteria which determines whether the number of cutpoints (or strong cutpoints) is finite or infinite almost surely.

This investigation was inspired by a result of James, Lyons and Peres (2008). They proved that for

$$p_i = \frac{1}{4i} + \frac{B}{4i \log i}, \quad i = 1, 2, \ldots, \ B > 1$$

the walk is transient and has only finitely many cutpoints.