Weak dependence and applications

Paul Doukhan

University Cergy-Pontoise

Among last years many efforts have been made to model weak dependence. The first successful idea was proposed in 1956 by Murray Rosenblatt and many authors like Pr Walter Philipp gave lots of improvements in this field. In 1984, a famous counterexample was enlighted by Donald Andrews; this shows that even if the idea is really a good one (see the main achievement of this idea in Rio's 2000 monograph). In fact many authors involved in the Philipp conference were pioneers like Bradley (who wrote a nice and huge definitive 3 volumes book). I was also one of those statisticians involved with mixing and reconsidering Andrews example and Association properties, I realized that something was perhaps missing. With Sana Louhichi I thus introduced a new version of weak dependence (developped in a volume coauthored with Dedecker, Lang, Leon, Louhichi and Prieur); we also prove that difficult models as Random systems with infinite connections from Iosifescu and Teodorescu also fit our definitions.

My talk will thus recall some main features and examples for this notion. After this, we shall recall the questions addressed by a main problem, estimation of a limit variance. This question was considered by several authors and beside the idea that this parameter is the value at 0 of a spectral density, several authors also considered the question, among them Peligrad and Shao for ρ-mixing processes; with Leon and Jakubowicz we consider the weakly dependent case. Besides the standard use to confidence bounds or self-normalized CLT we shall address the problem of Kolmogorov-Smirnov type tests for dependent sequences. Moreover, several examples are related with some explicit but definitely complicated limit variances, in such cases too this may be better to subsample this expression.