A central limit theorem for one-dimensional random walk on random scenery

George Deligiannidis

University of Nottingham

Let $X, X_1, X_2, \dots$ be iid, $\mathbb{Z}$-valued random variables whose characteristic function satisfies

$$f(t) = E[e^{itX}] = 1 - \gamma|t| + R(t), \qquad t \in [-\pi, \pi),$$

where $R(r) = o(|t|)$. Suppose that $S_n = \sum_{i=1}^n X_i$, that $\xi(\alpha), \alpha \in \mathbb{Z}$ are iid real-valued with mean zero and finite positive variance σ^2, and that $Z_n = \sum_{i=1}^n \xi(S_i)$. It is shown that $Z_n/\sqrt{n \log n}$ satisfies a central limit theorem and in particular that the laws of

$$Y_n(t) = \sqrt{\pi\gamma} Z_{[nt]}/\sqrt{2n \log n}, \quad t \in [0, 1]$$

converge weakly in $D[0, 1]$ to the Wiener measure.