

A strictly stationary, N -tuplewise independent counterexample to the central limit theorem

Richard Bradley

Indiana University

This is joint work with Alexander Pruss. For any given integer $N \geq 2$, there exists a strictly stationary ergodic sequence $(X_k, k \in \mathbf{Z})$ of random variables with the following properties: (i) the random variable X_0 is uniformly distributed on the interval $[-3^{1/2}, 3^{1/2}]$ (and hence $EX_0 = 0$ and $EX_0^2 = 1$); (ii) for every choice of N distinct integers $k(1), k(2), \dots, k(N)$, the random variables $X_{k(1)}, X_{k(2)}, \dots, X_{k(N)}$ are independent; (iii) the random variables $|X_k|, k \in \mathbf{Z}$ are independent; and (iv) for every infinite set $Q \subset \mathbf{N}$, there exist an infinite set $T \subset Q$ and a nondegenerate, non-normal probability measure μ on \mathbf{R} such that $(X_1 + X_2 + \dots + X_n)/n^{1/2}$ converges to μ in distribution as $n \rightarrow \infty, n \in T$. This example is a modification of a somewhat similar, nonstationary, N -tuplewise independent, identically distributed counterexample in Pruss [PTRF 111 (1998) 323-332]. It complements the strictly stationary, pairwise independent counterexamples in Janson [Stochastics 23 (1988) 439-448], and the strictly stationary, triplewise independent counterexamples developed in Bradley [PTRF 81 (1989) 1-10 and Rocky Mountain J. Math. 37 (2007) 25-44].