Flexible Regression and Smoothing"

Packages, Diagnostics and Algorithms

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gamlss

1. The R packages
2. The (gamlss) package and the `gamlss()` function
3. Diagnostics
   - Normalised quantile residuals
   - Worm plots
4. Algorithms
5. End

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The R packages

gamlss	the original package
gamlss.dist	all `gamlss.family` distributions
gamlss.data	different sets of data
gamlss.add	for extra additive terms
gamlss.cens	for censored (left, right or interval) response variables
gamlss.demo	demos for distributions and smoothing
gamlss.nl	non-linear term fitting
gamlss.tr	generating truncated distributions
gamlss.mx	finite mixtures distributions and random effects
gamlss.spatial	for spatial models
gamlss.util	for extra utilities

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The gamlss() function

```
gamlss( formula = ~1, sigma.formula= ~1,
      nu.formula = ~1, tau.formula = ~1,
      family = NO(),
      data = sys.parent(), weights = NULL,
      contrasts = NULL, method = RS(),  start.from = NULL,
      mu.start = NULL, sigma.start = NULL,
      nu.start = NULL, tau.start = NULL,
      mu.fix = FALSE, sigma.fix = FALSE, nu.fix = FALSE,
      tau.fix = FALSE, control = gamlss.control(...),
        i.control = glim.control(...), ...)
```

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Arguments of the gamlss() function

$$
\begin{aligned}
\text{formula} &= y \sim x1 + x3 \\
\text{sigma.fo} &= \sim x1 \\
\text{nu.fo} &= \sim x2 \\
\text{tau.fo} &= \sim 1 \\
\text{data} &= \text{abdom} \\
\text{family} &= \text{LO} \\
\text{weights} &= \text{freq} \\
\text{method} &= \text{mixed(10,50)} \\
\text{control} &= \text{gamlss.control(trace=FALSE)}
\end{aligned}
$$

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Starting values

Generally are not needed:

`start.mu= 2` or `start.mu=fitted(m1, "mu")`

The same applies for other parameters
`start.sigma` , `start.nu`, `start.tau`

Starting from a previous model

`start.from= model1`

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The gamlss package

Available functions

- Fitting or Updating a Model
- Extracting Information from the Fitted Model
- Selecting a Model
- Plotting and Diagnostics
- Centile Estimation

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Fitting or Updating a Model

`gamlss()`	for fitting and creating a `gamlss` object
`refit()`	to refit a `gamlss` object (i.e. continue iterations)
`update()`	to update a given `gamlss` model object
`gamlssML()`	fitting a parametric distribution to a single (response) variable
`histDist()`	to fit and plot a parametric distribution
`fitDist()`	select a parametric distribution from an appropriate list

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Extracting Information from the Fitted Model

`GAIC()`	generalised Akaike information criterion (or AIC)
`coef()`	the linear coefficients
`deviance()`	the global deviance $-2 \log L$
`fitted()`	the fitted values for a distribution parameter
`predict()`	to predict from new data individual distribution parameter values
`predictAll()`	to predict from new data all the distribution parameter values
`print()`	: to print a `gamlss` object
`residuals()`	to extract the normalised (randomised) quantile residuals
`summary()`	to summarise the fit in a `gamlss` object
`vcov()`	to extract the variance-covariance matrix of the beta estimates.

Selecting a Model

`add1()` `drop1()` to add or drop a single term

`stepGAIC()` to select explanatory terms in one parameter

`stepGAICAll.A()` to select explanatory terms in all the parameters (strategy A)

`stepGAICAll.B()` to select explanatory terms in all the parameters (strategy B)

`gamlssCV()` to evaluate a model using cross valdation

`add1TGD()` `drop1TGD()` to add or drop a single term using a validation or test data set

`stepTGD()` selecting variables using a test data set the global deviance for new (test) data set given a fitted gamlss model.

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Diagnostics

`plot()`	a plot of four graphs for the normalized (randomized) quantile residuals
`pdf.plot()`	for plotting the pdf functions for a given fitted `gamlss` object or a given `gamlss.family` distribution
`Q.stats()`	for printing and plotting the Q statistics of Royston and Wright (2000).
`rqres.plot()`	for plotting QQ-plots of different realisations of randomised residuals (for discrete distributions)
`wp()`	worm plot of the residuals from a fitted `gamlss` object
`dtop()`	detrended Own's plot of the residuals

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Centile estimation

`lms()` a function trying to automate the process of fitting growth curves

`centiles()` to plot centile curves against an x-variable.

`centiles.com()` to compare centiles curves for more than one `object`.

`centiles.split()` as for `centiles()`, but splits the plot at specified values of x.

`centiles.pred()` to predict and plot centile curves for new x-values.

`centiles.fan()` fan plot od centile curves

`fitted.plot()` to plot fitted values for all the parameters against an x-variable

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Other useful functions

`prof.dev()` the profile global deviance of one of the distribution parameters

`prof.term()` for plotting the profile global deviance of one of the model (beta) parameters

`show.link()` for showing available link functions

`term.plot()` for plotting additive (smoothing) terms

`gen.likelihood()` generates the likelihood from a GAMLSS fitted model [used in vcov()]

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Normalised quantile residuals-continuous response

If y_i is an observation from a continuous response variable then

$$\hat{u}_i = F(y_i|\hat{\boldsymbol{\theta}}_i)$$

where $F(y_i|\boldsymbol{\theta}_i)$ is the assumed cumulative distribution function for case i.

$$r_i = \Phi^{-1}(u_i)$$

So r_i will have an approximately standard normal distribution

$$r_i = \text{"z-scores"}$$

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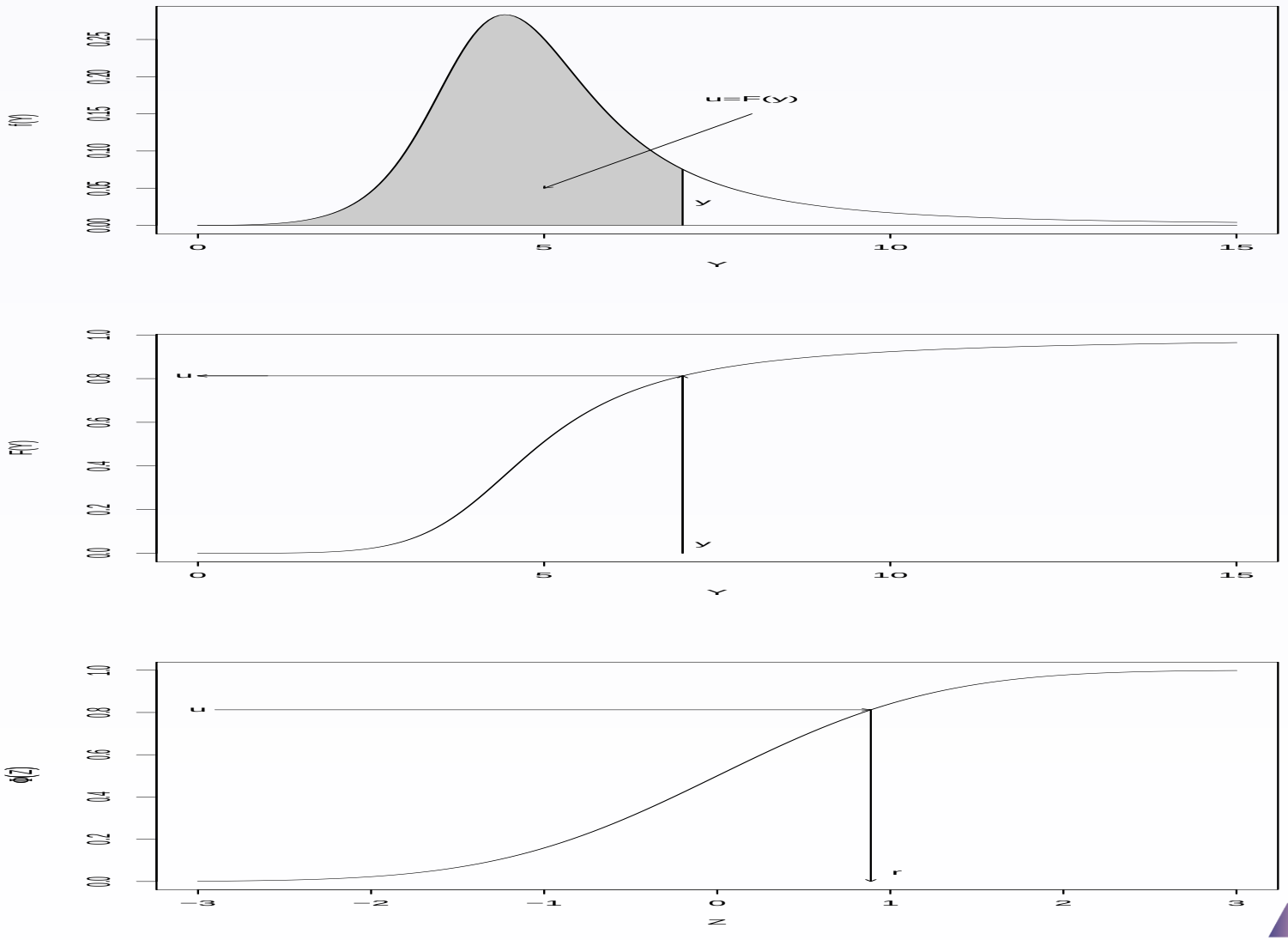
Normalised quantile residuals-continuous response



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Normalised quantile residuals-discrete response

If y_i is an observation from a discrete integer response then

$$\hat{u}_i$$

is a random value from the uniform distribution on the interval

$$[u_1, u_2] = \left[F(y_i - 1|\hat{\boldsymbol{\theta}}_i), F(y_i|\hat{\boldsymbol{\theta}}_i)\right]$$

$$\hat{r}_i = \Phi^{-1}(\hat{u}_i)$$

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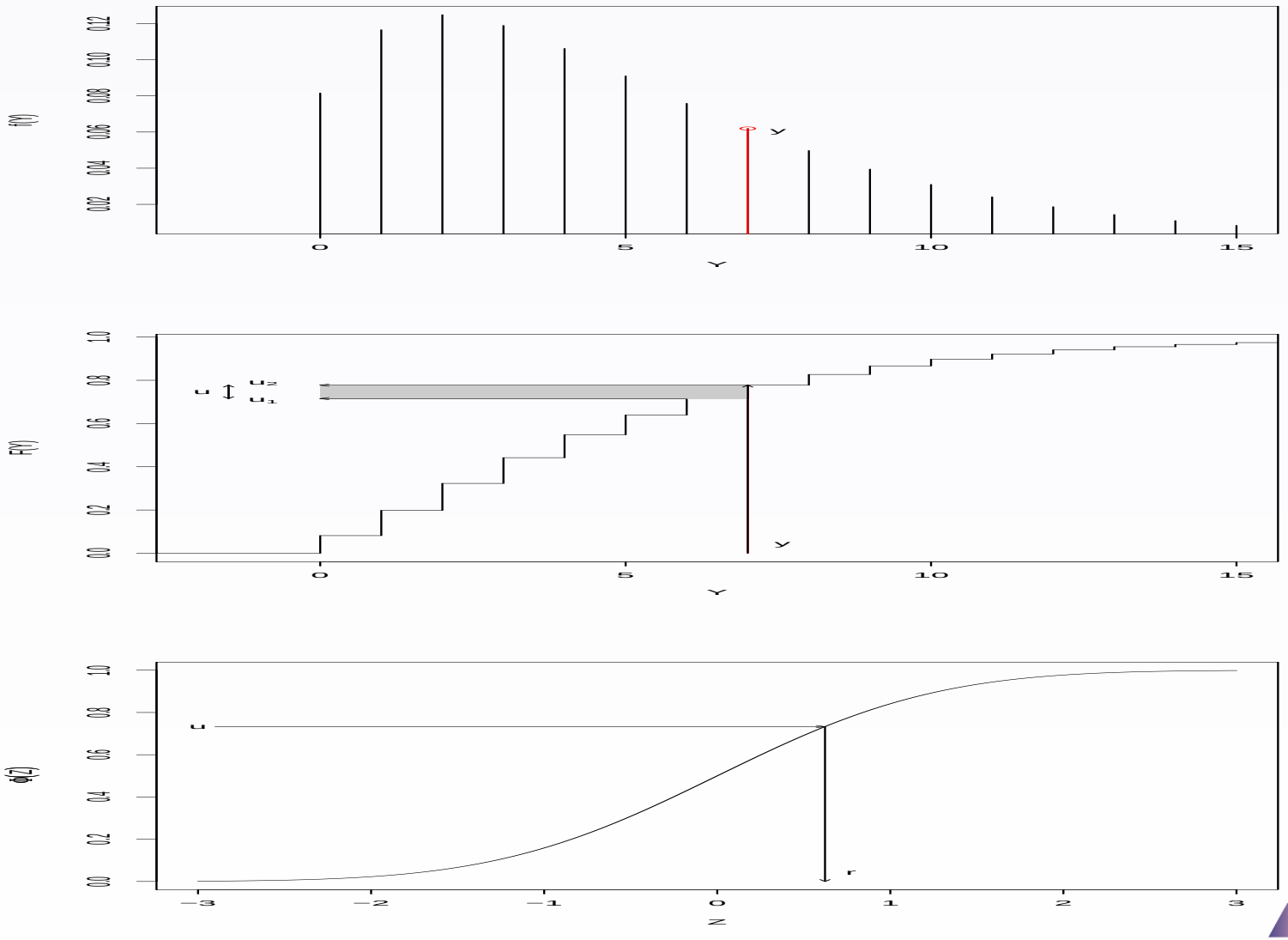
Normalised quantile residuals-continuous response



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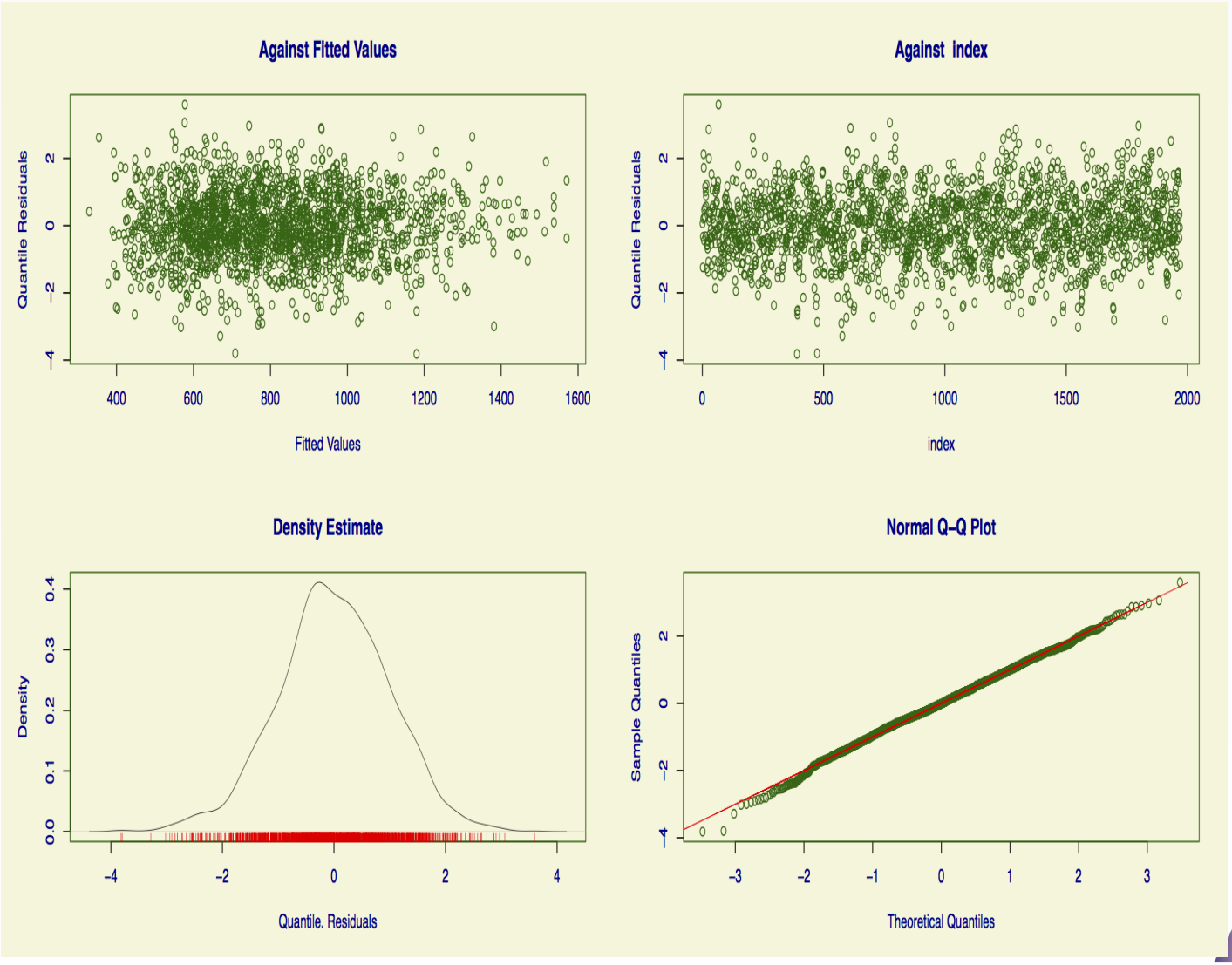
using plot()



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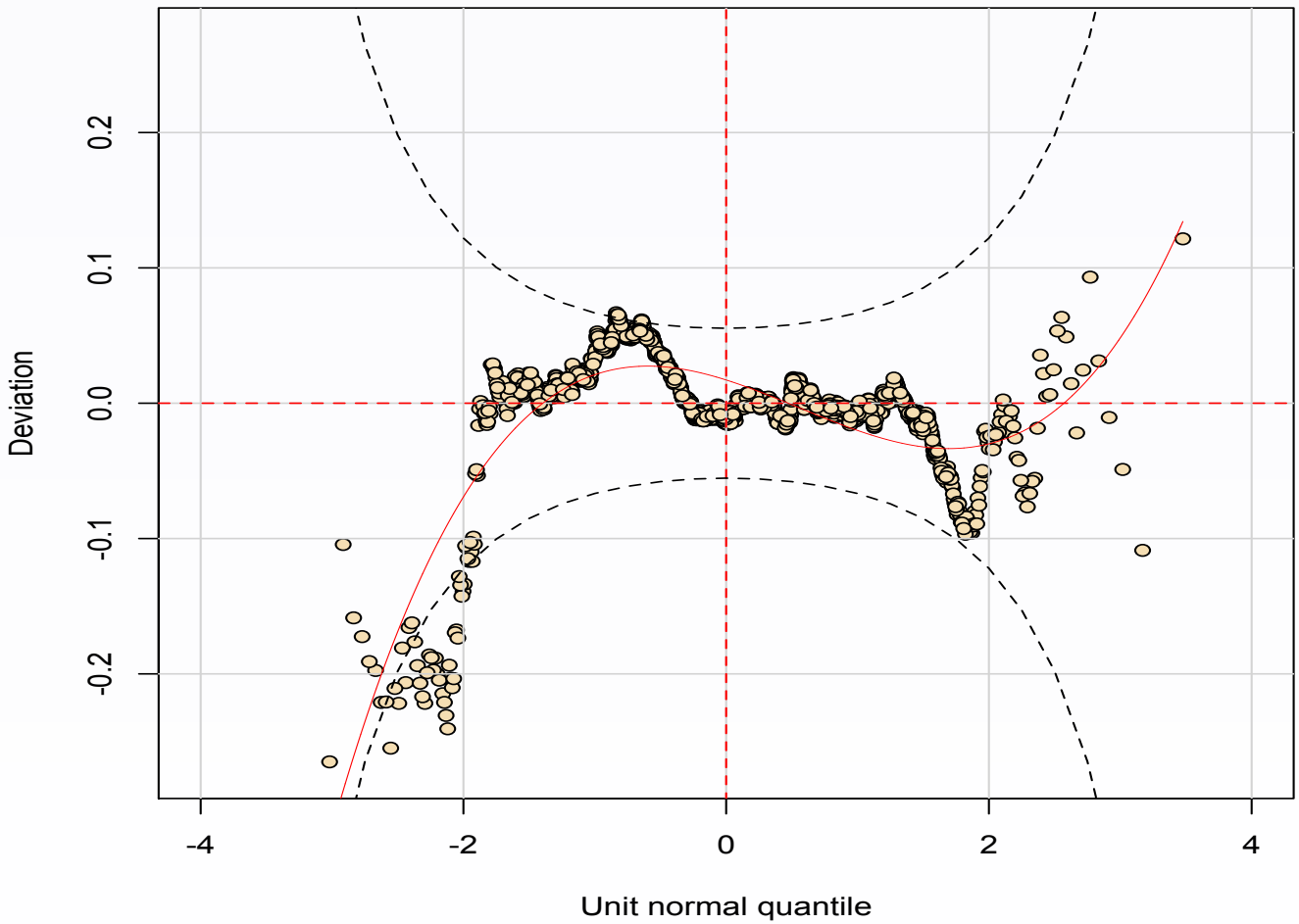
Worm plots: using wp() van Buuren and Fredriks [2001]



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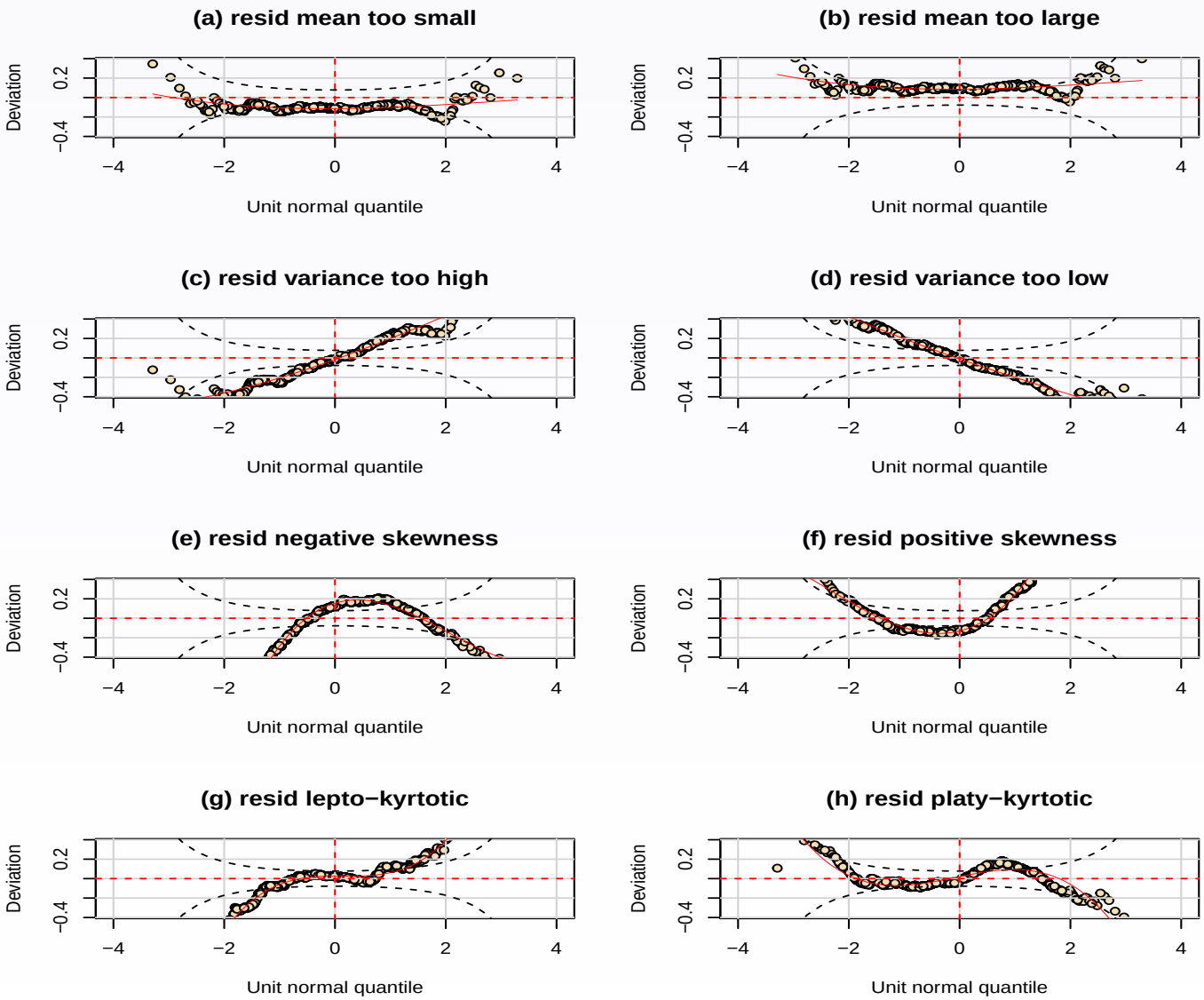
Worm plots: different types



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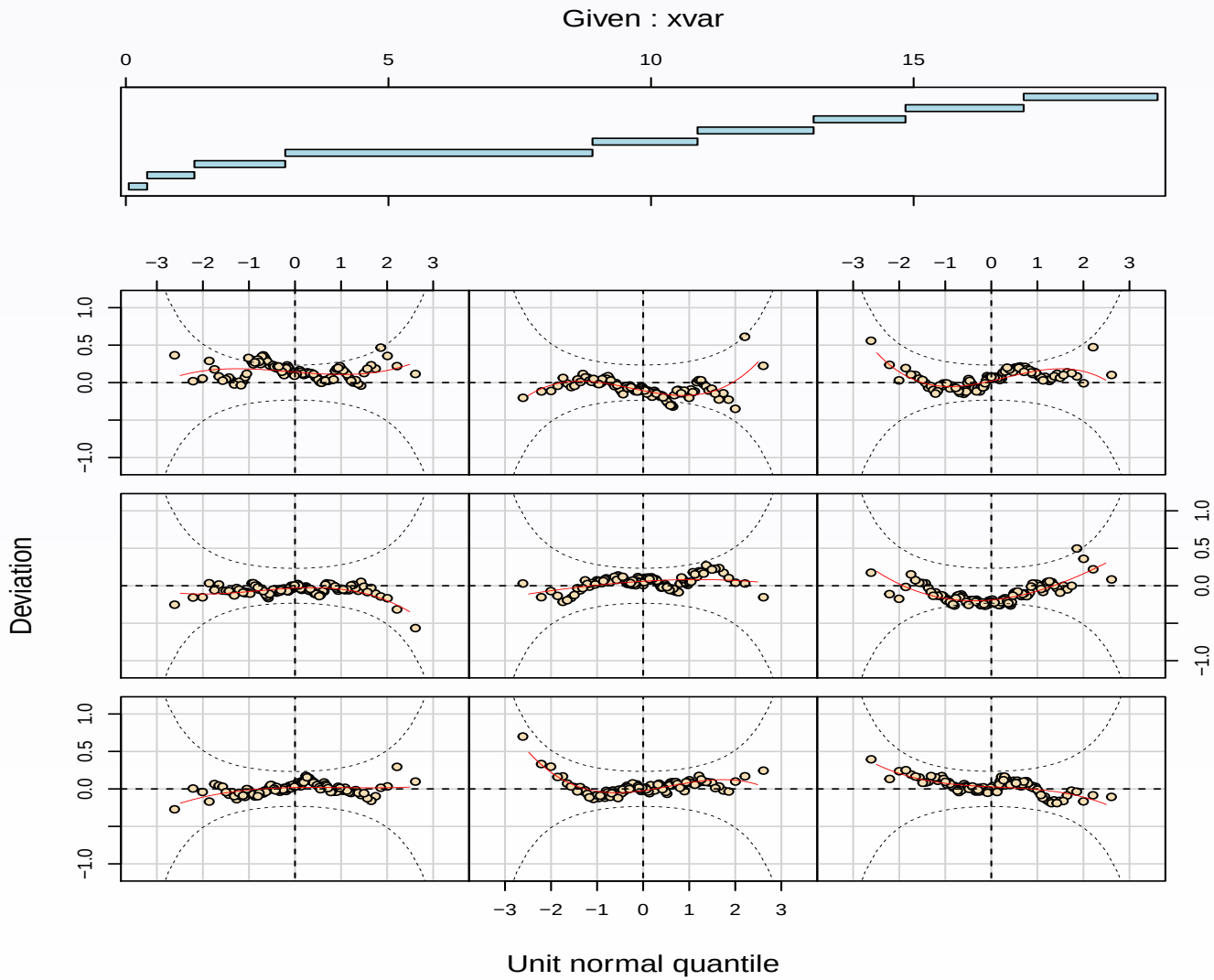
Multiple worm plots against different range of x-variable



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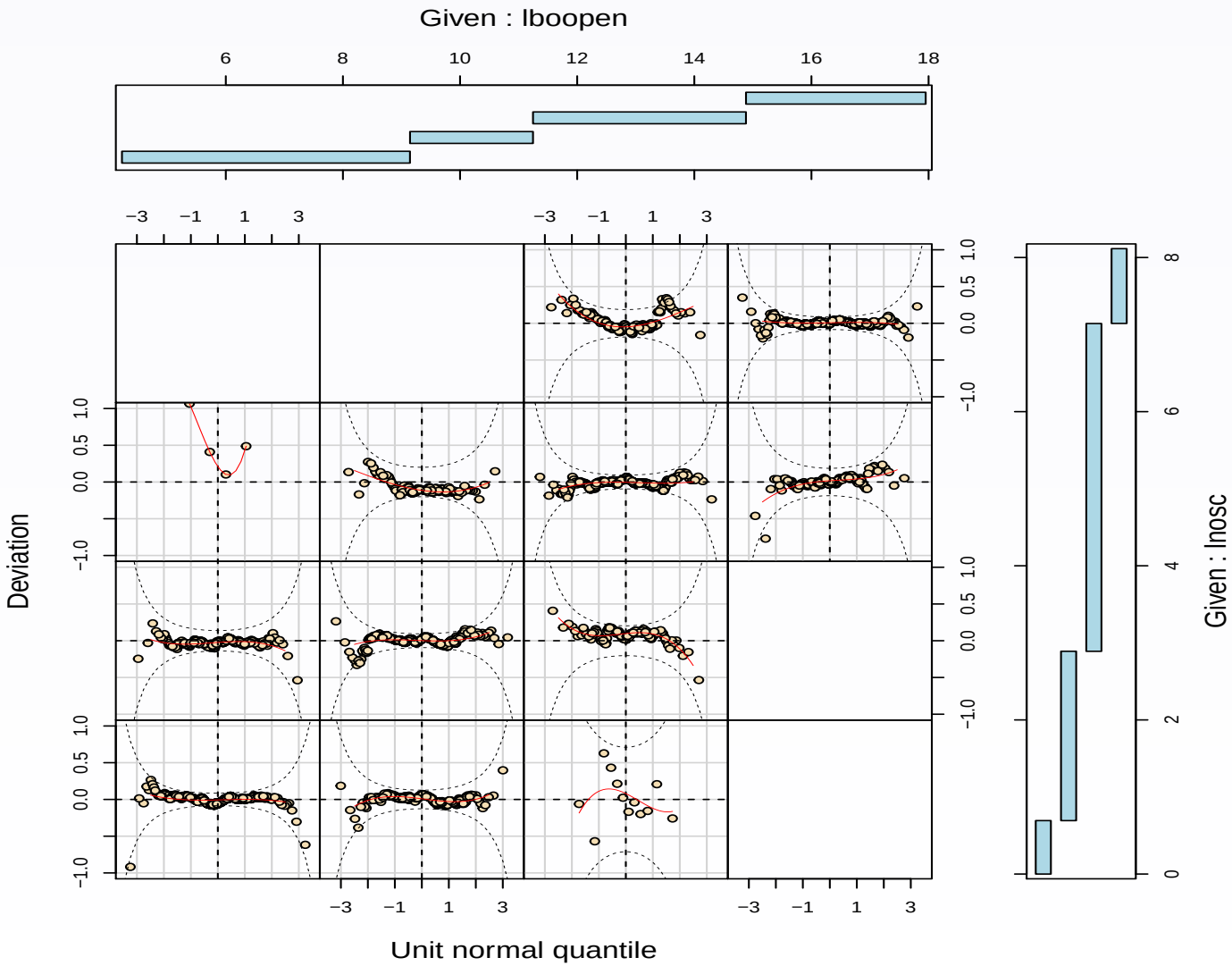
Multiple worm plots against different range of two x-variables



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Worm plot conclusion

Worm plot are useful in:

- checking the assumed distribution of the response variable,
- checking whether the distribution is fitted correctly in all part of explanatory variable(s)

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Algorithms: GAMLSS model

The model

$$
\begin{aligned}
\mathbf{y} &\overset{\text{ind}}{\sim} D(\boldsymbol{\mu}, \boldsymbol{\sigma}, \boldsymbol{\nu}, \boldsymbol{\tau}) \\
g_1(\boldsymbol{\mu}) &= \mathbf{X}_1\boldsymbol{\beta}_1 + s_{11}(\mathbf{x}_{11}) + \ldots + s_{1J_1}(\mathbf{x}_{1J_1}) \\
g_2(\boldsymbol{\sigma}) &= \mathbf{X}_2\boldsymbol{\beta}_2 + s_{21}(\mathbf{x}_{21}) + \ldots + s_{2J_2}(\mathbf{x}_{2J_2}) \\
g_3(\boldsymbol{\nu}) &= \mathbf{X}_3\boldsymbol{\beta}_3 + s_{31}(\mathbf{x}_{31}) + \ldots + s_{3J_3}(\mathbf{x}_{3J_3}) \\
g_4(\boldsymbol{\tau}) &= \mathbf{X}_4\boldsymbol{\beta}_4 + s_{41}(\mathbf{x}_{41}) + \ldots + s_{4J_4}(\mathbf{x}_{4J_4})
\end{aligned}
$$

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Algorithms: GAMLSS model random effects

$$
\begin{aligned}
\mathbf{y} &\overset{\text{ind}}{\sim} D(\boldsymbol{\mu}, \boldsymbol{\sigma}, \boldsymbol{\nu}, \boldsymbol{\tau}) \\
g_1(\boldsymbol{\mu}) &= \mathbf{X}_1\boldsymbol{\beta}_1 + \mathbf{Z}_{11}\boldsymbol{\gamma}_{11} + \ldots + \mathbf{Z}_{1k_1}\boldsymbol{\gamma}_{1J_1} \\
g_2(\boldsymbol{\sigma}) &= \mathbf{X}_2\boldsymbol{\beta}_2 + \mathbf{Z}_{21}\boldsymbol{\gamma}_{21} + \ldots + \mathbf{Z}_{2k_2}\boldsymbol{\gamma}_{2J_2} \\
g_3(\boldsymbol{\nu}) &= \mathbf{X}_3\boldsymbol{\beta}_3 + \mathbf{Z}_{31}\boldsymbol{\gamma}_{31} + \ldots + \mathbf{Z}_{3k_3}\boldsymbol{\gamma}_{3J_3} \\
g_4(\boldsymbol{\tau}) &= \mathbf{X}_4\boldsymbol{\beta}_4 + \mathbf{Z}_{41}\boldsymbol{\gamma}_{41} + \ldots + \mathbf{Z}_{4k_4}\boldsymbol{\gamma}_{4J_4}, \\
\boldsymbol{\gamma}_{kJ} &\sim N(\mathbf{0}, \lambda_{kJ}^{-1}\mathbf{G}^{-1})
\end{aligned}
\tag{1}
$$

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Algorithms: Estimation

we will need estimates for the 'betas',

$$\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{\beta}_3, \boldsymbol{\beta}_4)$$

the 'gammas'

$$\boldsymbol{\gamma} = (\boldsymbol{\gamma}_{11}, \ldots, \boldsymbol{\gamma}_{1J_1}, \boldsymbol{\gamma}_{21}, \ldots, \boldsymbol{\gamma}_{4J_4}),$$

and the 'lambdas'

$$\boldsymbol{\lambda} = (\boldsymbol{\lambda}_{11}, \ldots, \boldsymbol{\lambda}_{1J_1}, \boldsymbol{\lambda}_{21}, \ldots, \boldsymbol{\lambda}_{4J_4}).$$

For parametric models: ML estimators for $\boldsymbol{\beta}$
For smoothing (random effect models) : MAP estimators for $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ for fixed $\boldsymbol{\lambda}$

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Algorithms: posterior probability

$$
\begin{aligned}
f(\boldsymbol{\beta}, \boldsymbol{\gamma}, |\mathbf{y}, \boldsymbol{\lambda}) &\propto f(\mathbf{y}|\boldsymbol{\beta}, \boldsymbol{\gamma}) f(\boldsymbol{\gamma}|\boldsymbol{\lambda}) \\
&\propto \underbrace{L(\boldsymbol{\beta}, \boldsymbol{\gamma})}_{\text{Likelihood}} \underbrace{\prod_k \prod_j f(\gamma_{kj}|\lambda_{kj})}_{\text{prior for } \boldsymbol{\gamma}},
\end{aligned}
$$

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Algorithms: penalised likelihood

$$
\begin{aligned}
\log f(\boldsymbol{\beta}, \boldsymbol{\gamma}, |\mathbf{y}, \boldsymbol{\lambda}) &= \underbrace{\ell(\boldsymbol{\beta}, \boldsymbol{\gamma})}_{\text{log-likelihood}} + \sum_k \sum_j \log f(\gamma_{kj}|\lambda_{kj}) + \underbrace{c(\mathbf{y}, \boldsymbol{\lambda})}_{\text{constant}} \\
&= \underbrace{\ell_h(\boldsymbol{\beta}, \boldsymbol{\gamma}, |\boldsymbol{\lambda})}_{\text{hierarchical log-likelihood}} + \underbrace{c(\mathbf{y}, \boldsymbol{\lambda})}_{\text{constant}} \\
&= \underbrace{\ell(\boldsymbol{\beta}, \boldsymbol{\gamma})}_{\text{log-likelihood}} \underbrace{-\frac{1}{2} \sum_k \sum_j \lambda_{kj} \boldsymbol{\gamma}_{kj}^\top \mathbf{G}_{kj} \boldsymbol{\gamma}_{kj}}_{\text{log-normal exponent}} + \underbrace{c_1(\mathbf{y}, \boldsymbol{\lambda})}_{\text{constant}} \\
&= \underbrace{\ell_p(\boldsymbol{\beta}, \boldsymbol{\gamma}, |\boldsymbol{\lambda})}_{\text{penalised log-likelihood}} + \underbrace{c_1(\mathbf{y}, \boldsymbol{\lambda})}_{\text{constant}}
\end{aligned}
$$

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Algorithms: Estimating β and γ for fixed λ

RS a generalisation of the MADAM algorithm, Rigby and Stasinopoulos (1996a)

CG a generalisation of Cole and Green (1992) algorithm

mixed a mixture of RS+CG (i.e. j iterations of RS, followed by k iterations of CG)

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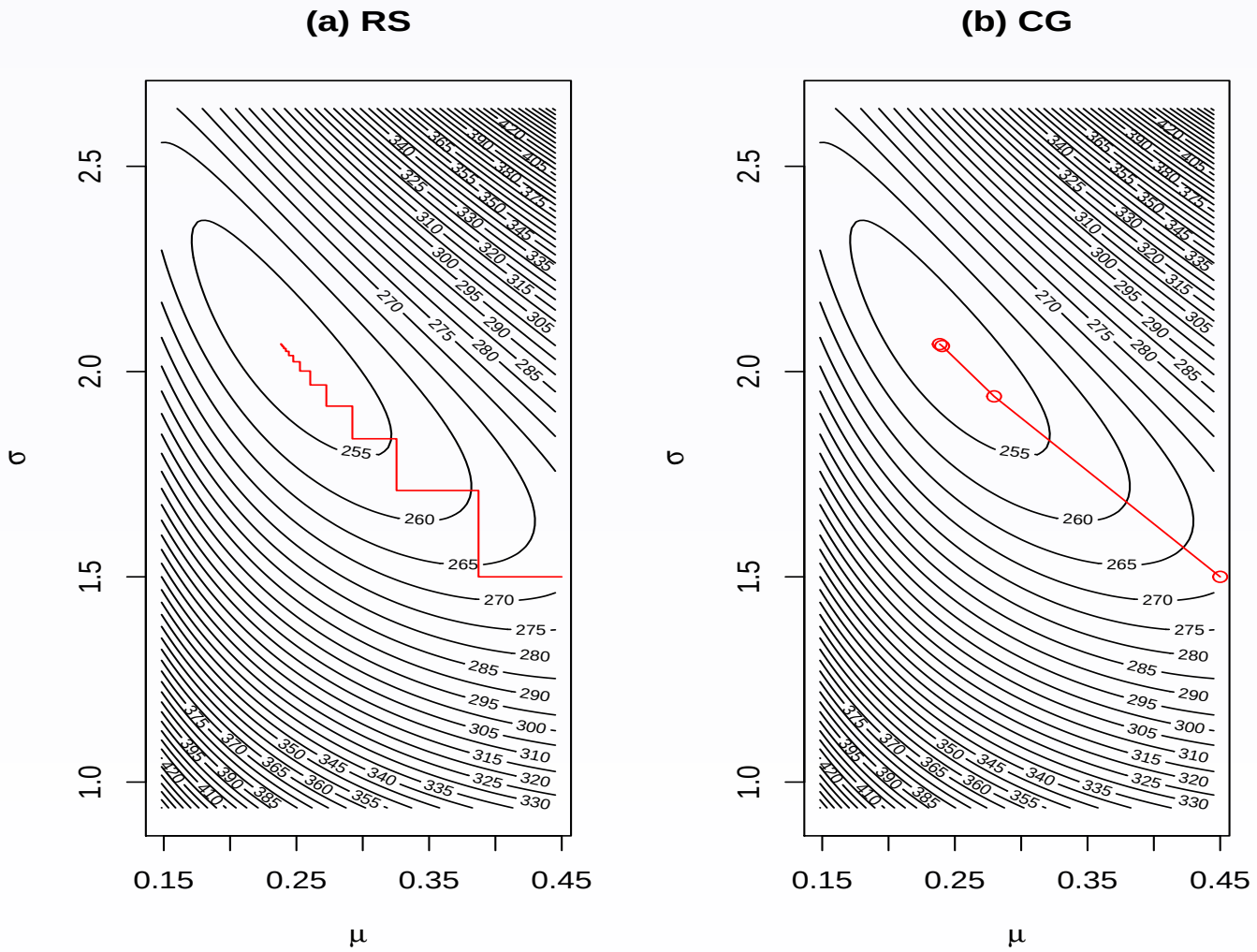
Algorithms



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Algorithms: The RS algorithm

- the *outer iteration*
- the *inner iteration* (or local scoring or GLIM algorithm)
- the *modified backfitting* algorithm

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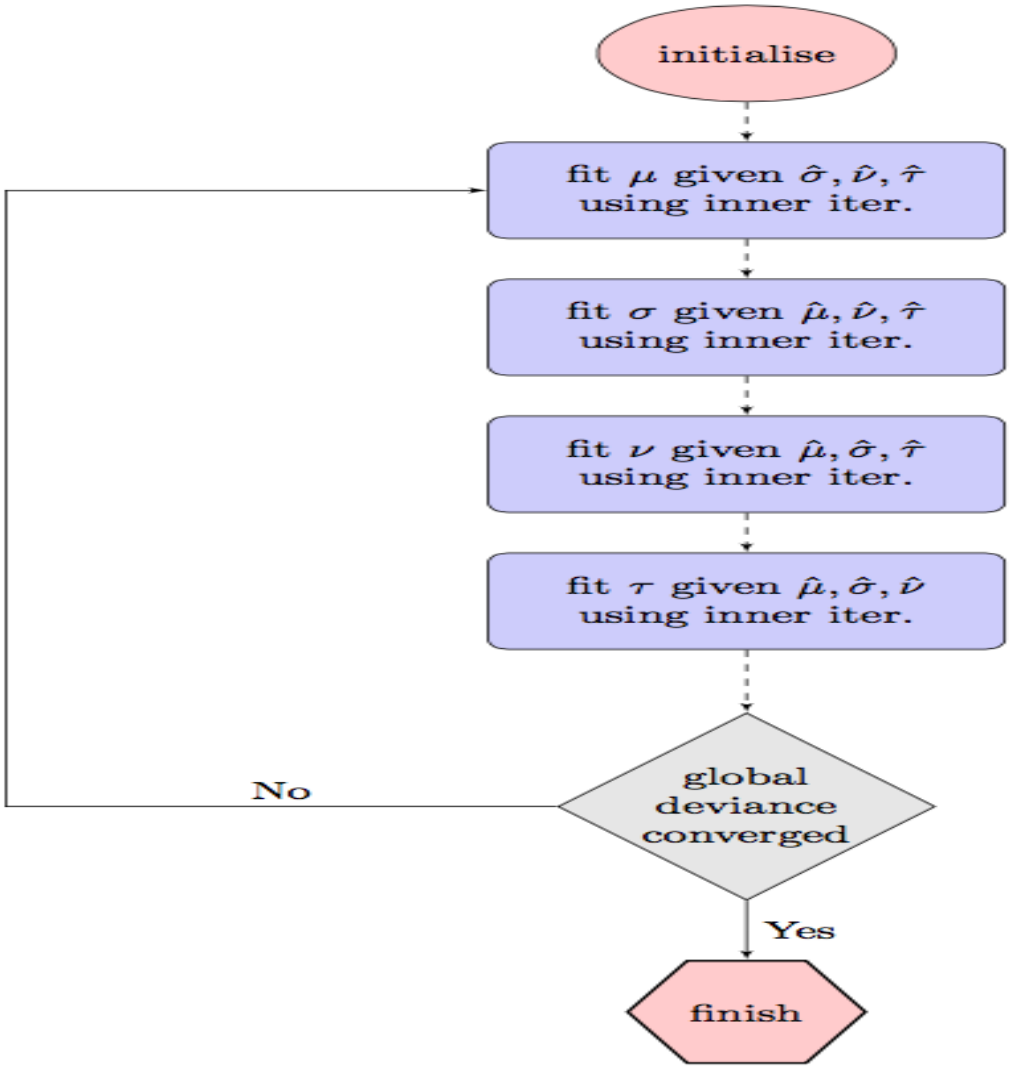
Algorithms: outer iteration



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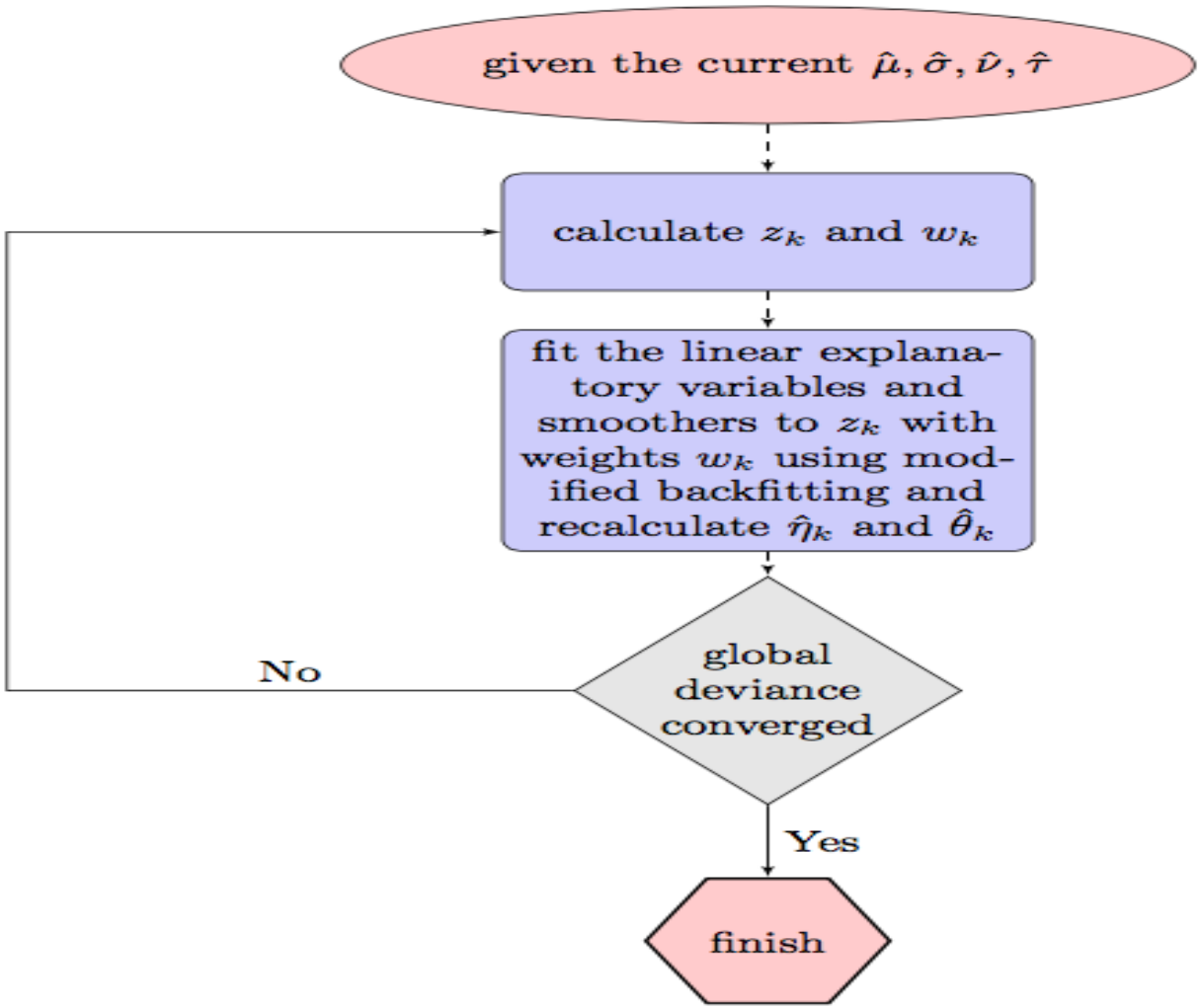
Algorithms: inner iteration



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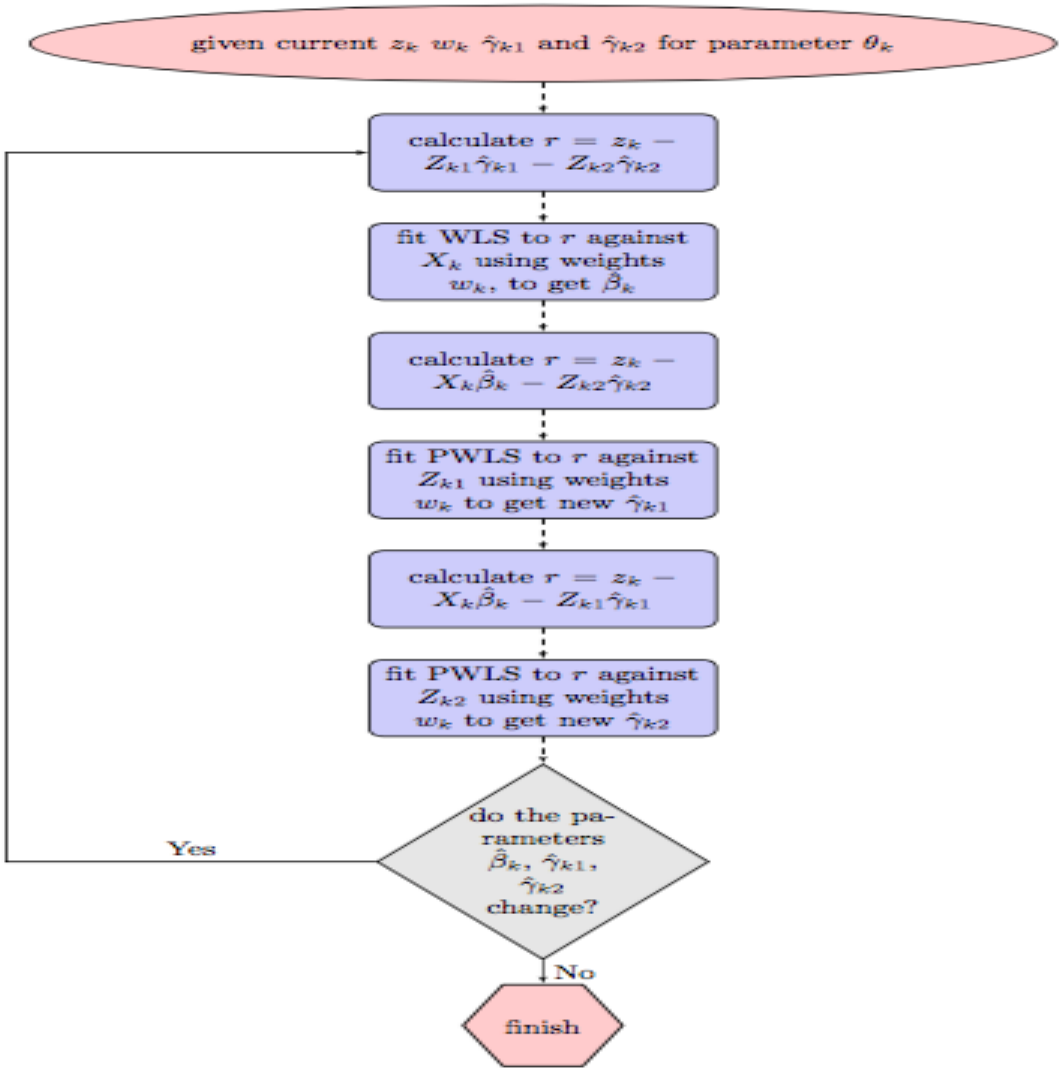
Algorithms: modified backfitting



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Algorithms: properties

Advantages of algorithms

1. flexible modular fitting procedure
2. easy implementation of new distributions
3. easy implementation of new additive terms
4. simple starting values for (μ, σ, ν, τ) easily found
5. stable and reliable algorithms
6. fast fitting (for fixed hyperparameters)

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Algorithms: Estimating λ

locally: when the method of estimation of each λ_{kj} is applied each time within the backfitting algorithm

globally: when the method is applied outside the `RS` or `CG` GAMLSS algorithm.

Different methodologies for estimating the smoothing hyper-parameters:

- Generalised cross validation (GCV),
- Generalised Akaike information criterion (GAIC), and
- Maximum likelihood based methods (ML/REML).

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END

for more information see

www.gamlss.org

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