Mathematische Statistik
Gebräuchliche Verteilungen

Diskrete Verteilungen:

• **Bernoulli**(p)
$P(X = x|p) = p^x(1-p)^{1-x}; \quad x = 0,1; \quad 0 \leq p \leq 1$
$\mathrm{E}(X) = p, \quad \mathrm{var}(X) = p(1-p)$
$M_X(t) = (1-p) + pe^t$

• **Binomial**(n, p)
$P(X = x|n, p) = \binom{n}{x} p^x(1-p)^{n-x}; \quad x = 0, 1, \ldots, n; \quad 0 \leq p \leq 1$
$\mathrm{E}(X) = np, \quad \mathrm{var}(X) = np(1-p)$
$M_X(t) = [(1-p) + pe^t]^n$

• **Discrete Uniform**(N)
$P(X = x|N) = 1/N; \quad x = 1, 2, \ldots, N; \quad N = 1, 2, \ldots$
$\mathrm{E}(X) = (N+1)/2, \quad \mathrm{var}(X) = (N+1)(N-1)/12$
$M_X(t) = 1/N \sum_{i=1}^{N} e^{it}$

• **Geometric**(p)
$P(X = x|p) = p(1-p)^{x-1}; \quad x = 1, 2, \ldots; \quad 0 \leq p \leq 1$
$\mathrm{E}(X) = 1/p, \quad \mathrm{var}(X) = (1-p)/p^2$
$M_X(t) = pe^t/[1 - (1-p)e^t]; \quad t < -\log(1-p)$

• **Hypergeometric**(N, M, K)
$P(X = x|N, M, K) = \binom{M}{x}\binom{N-M}{K-x}/\binom{N}{K}; \quad x = 0, 1, \ldots, K; \; M - (N-K) \leq x \leq M;$
$N, M, K \geq 0$
$\mathrm{E}(X) = KM/N, \quad \mathrm{var}(X) = KM/N \cdot (N-M)(N-K)/[N(N-1)]$

• **Negative-Binomial**(r, p)
$P(X = x|r, p) = \binom{r+x-1}{x} p^r(1-p)^x; \quad x = 0, 1, \ldots; \quad 0 \leq p \leq 1$
$\mathrm{E}(X) = r(1-p)/p, \quad \mathrm{var}(X) = r(1-p)/p^2$
$M_X(t) = \{p/[1 - (1-p)e^t]\}^r; \quad t < -\log(1-p)$

• **Poisson**(λ)
$P(X = x|\lambda) = e^{-\lambda}\lambda^x/x!; \quad x = 0, 1, \ldots; \quad 0 \leq \lambda < \infty$
$\mathrm{E}(X) = \lambda, \quad \mathrm{var}(X) = \lambda$
$M_X(t) = e^{\lambda(e^t - 1)}$

Stetige Verteilungen:

• **Beta**(α,β)
$f(x|\alpha,\beta)=x^{\alpha-1}(1-x)^{\beta-1}/B(\alpha,\beta);\quad 0\le x\le 1;\quad \alpha,\beta>0$
$\mathrm{E}(X)=\alpha/(\alpha+\beta),\quad \mathrm{var}(X)=\alpha\beta/[(\alpha+\beta)^2(\alpha+\beta+1)]$
$M_X(t)=1+\sum_{k=1}^{\infty}\left[\prod_{r=0}^{k-1}(\alpha+r)/(\alpha+\beta+r)\right]t^k/k!$

• **Cauchy**(θ,σ)
$f(x|\theta,\sigma)=[\pi\sigma(1+[(x-\theta)/\sigma]^2)]^{-1};\quad x,\theta\in\mathbb{R};\quad \sigma>0$
$\mathrm{E}(X)$, $\mathrm{var}(X)$, sowie $M_X(t)$ existieren nicht

• **Chi-Squared**(p)
$f(x|p)=x^{(p/2)-1}e^{-x/2}/[\Gamma(p/2)2^{p/2}];\quad 0\le x<\infty;\quad p=1,2,\ldots$
$\mathrm{E}(X)=p,\quad \mathrm{var}(X)=2p$
$M_X(t)=\left[1/(1-2t)\right]^{p/2};\quad t<1/2$

• **Double Exponential**(μ,σ) (Laplace)
$f(x|\mu,\sigma)=e^{-|x-\mu|/\sigma}/(2\sigma);\quad x,\mu\in\mathbb{R};\quad \sigma>0$
$\mathrm{E}(X)=\mu,\quad \mathrm{var}(X)=2\sigma^2$
$M_X(t)=e^{\mu t}/[1-(\sigma t)^2];\quad |t|<1/\sigma$

• **Exponential**(β)
$f(x|\beta)=e^{-x/\beta}/\beta;\quad 0\le x<\infty;\quad \beta>0$
$\mathrm{E}(X)=\beta,\quad \mathrm{var}(X)=\beta^2$
$M_X(t)=1/(1-\beta t);\quad t<1/\beta$

• **F**(p,q)
$f(x|p,q)=1/B(p/2,q/2)\cdot\left(\frac{p}{q}\right)^{p/2}x^{(p-2)/2}/\left(1+x(p/q)\right)^{(p+q)/2};\quad 0\le x<\infty;\quad p,q=1,\ldots$
$\mathrm{E}(X)=q/(q-2),\ q>2;$
$\mathrm{var}(X)=2\left[q/(q-2)\right]^2(p+q-2)/[p(q-4)],\ q>4$
$M_X(t)$ existiert nicht

• **Gamma**(α,β)
$f(x|\alpha,\beta)=x^{\alpha-1}e^{-x/\beta}/[\Gamma(\alpha)\beta^\alpha];\quad 0\le x<\infty;\quad \alpha,\beta>0$
$\mathrm{E}(X)=\alpha\beta,\quad \mathrm{var}(X)=\alpha\beta^2$
$M_X(t)=\left(1/(1-\beta t)\right)^\alpha;\quad t<1/\beta$

• **Logistic**(μ,β)
$f(x|\mu,\beta)=1/\beta\cdot e^{-(x-\mu)/\beta}/[1+e^{-(x-\mu)/\beta}]^2;\quad x,\mu\in\mathbb{R};\quad \beta>0$
$\mathrm{E}(X)=\mu,\quad \mathrm{var}(X)=\pi^2\beta^2/3$
$M_X(t)=e^{\mu t}\Gamma(1-\beta t)\Gamma(1+\beta t);\quad |t|<1/\beta$

• **Lognormal**(μ,σ^2)
$f(x|\mu,\sigma^2)=1/\sqrt{2\pi\sigma^2}\cdot e^{-(\log x-\mu)^2/(2\sigma^2)}/x;\quad 0\le x<\infty;\quad \mu\in\mathbb{R};\quad \sigma>0$
$\mathrm{E}(X)=e^{\mu+(\sigma^2/2)},\quad \mathrm{var}(X)=e^{2(\mu+\sigma^2)}-e^{2\mu+\sigma^2}$
$M_X(t)$ existiert nicht

• **Normal**(μ,σ^2) (Gauss)
$f(x|\mu,\sigma^2)=1/\sqrt{2\pi\sigma^2}\cdot e^{-(x-\mu)^2/(2\sigma^2)};\quad x,\mu\in\mathbb{R};\quad \sigma>0$
$\mathrm{E}(X)=\mu,\quad \mathrm{var}(X)=\sigma^2$
$M_X(t)=e^{\mu t+\sigma^2t^2/2}$

• **Pareto**(α, β)
$f(x|\alpha, \beta) = \beta\alpha^\beta / x^{\beta+1}; \quad \alpha < x < \infty; \quad \alpha, \beta > 0$
$\mathrm{E}(X) = \beta\alpha/(\beta - 1), \, \beta > 1$
$\mathrm{var}(X) = \beta\alpha^2/[(\beta-1)^2(\beta-2)], \, \beta > 2$
$M_X(t)$ existiert nicht

• $\mathbf{t}(p)$
$f(x|p) = \Gamma((p+1)/2)/\Gamma(p/2) \cdot 1/\sqrt{p\pi} \cdot [1 + (x^2/p)]^{-(p+1)/2}; \quad x \in \mathbb{R}; \quad p = 1, \ldots$
$\mathrm{E}(X) = 0, \quad \mathrm{var}(X) = p/(p-2), \, p > 2$
$M_X(t)$ existiert nicht

• **Uniform**(a, b)
$f(x|a, b) = 1/(b - a); \quad a \leq x \leq b$
$\mathrm{E}(X) = (b + a)/2, \quad \mathrm{var}(X) = (b - a)^2/12$
$M_X(t) = (e^{bt} - e^{at})/[(b - a)t]$

• **Weibull**(γ, β)
$f(x|\gamma, \beta) = \gamma x^{\gamma-1} e^{-x^\gamma/\beta}/\beta; \quad 0 \leq x < \infty; \quad \gamma, \beta > 0$
$\mathrm{E}(X) = \beta^{1/\gamma}\Gamma(1 + 1/\gamma), \quad \mathrm{var}(X) = \beta^{2/\gamma}\Big[\Gamma(1 + 2/\gamma) - \Gamma^2(1 + 1/\gamma)\Big]$
$M_X(t)$ existiert nur für $\gamma \geq 1$