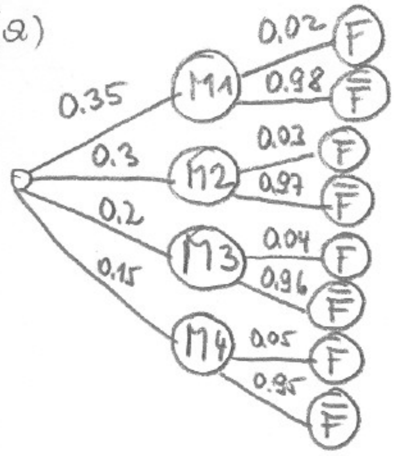
LÖSUNGSBLATT WTH u. STOCH. PROZESSE
28.8.2003

① (a)



(4P)

(b) $P(\text{fehlerhaft}) = P(F)$

$= 0{,}35 \cdot 0.02 + 0.3 \cdot 0.03 + 0.2 \cdot 0.04 + 0.15 \cdot 0.05 = \underline{0.0315}$ (6P)

(c) $P(M3|F) = \dfrac{P(F|M3)P(M3)}{P(F)} = \dfrac{0.2 \cdot 0.04}{0.0315} = \underline{0.254}$ (6P)

(d) $P(\text{mind. } 1 \text{ fh. Teil}) \geq 0.95$

$\Leftrightarrow (1-0.0315)^n < 0.05 \Rightarrow n \geq \dfrac{\log(0{,}05)}{\log(0.9685)} = \underline{93{,}6}$ (4P)

② (a) $X \sim B(n, \frac{\alpha}{360})$ (2P)

(b) $P(X=2) = \binom{10}{2} p^2 (1-p)^8 \longrightarrow \max. \Rightarrow f(p) = 45 p^2 (1-p)^8$

$f'(p) = 90 p (1-p)^8 - 180 p^2 (1-p)^7 \overset{!}{=} 0 \Leftrightarrow \underline{p = \frac{1}{5}}$ (8P)

(c) $\alpha = 90° \Rightarrow p = \frac{90}{360} = \frac{1}{4}$

$P_X(X \geq 1) \geq 0.95 \Leftrightarrow P_X(X=0) < 0.05 \Leftrightarrow \left(\frac{3}{4}\right)^n < 0.05 \Rightarrow \underline{n > 10.4}$ (4P)

(d) $p = \frac{1}{4}$, G = Gewinn

$G = k$	-2	1	8
$X = i$	0	1	2
p_i	9/16	6/16	1/16

$E(G) = -2 \times \frac{9}{16} + 1 \times \frac{6}{16} + 8 \times \frac{1}{16}$

$= -\frac{4}{16} = \underline{-\frac{1}{4}}$ unfair! (6P)

③ $X \sim N(60, 8.5)$

(a) $P_X(X<50) + P_X(X>80) = \Phi\Big(\underbrace{\tfrac{50-60}{8.5}}_{-1.176}\Big) + 1 - \Phi\Big(\underbrace{\tfrac{80-60}{8.5}}_{2.353}\Big) =$

$= 1 - \Phi(1.176) + 1 - \Phi(2.353) = 0{,}1198 + 0.0093 = \underline{0.129}$ (6P)

(b) $X \sim B(30, 0.035)$; $P_X(X \leq 2) = \sum_{i=0}^{2} \binom{30}{i} 0.035^i \times 0.965^{30-i} = \underline{0.9136}$ (10P)

(c) $\Phi\left(\frac{X-60}{8.5}\right) = 0.95 \Leftrightarrow \frac{X-60}{8.5} = 1.6445 \Leftrightarrow \underline{X = 73{,}98}$ (4P)

④ $f_{X,Y}(x,y) = \begin{cases} cxy & 0<x<1,\ 0<y<x \\ 0 & \text{sonst} \end{cases}$

(a) $\int_0^1\int_0^x cxy\,dy\,dx \overset{!}{=} 1 \Leftrightarrow c\int_0^1 x\left[\int_0^x y\,dy\right]dx \overset{!}{=} 1$

$\Leftrightarrow c\int_0^1 \frac{x^3}{2}\,dx \overset{!}{=} 1 \Leftrightarrow \frac{c}{8} \overset{!}{=} 1 \Leftrightarrow \underline{c=8}$ (4P)

(b) $f_X(x) = \int_0^x 8xy\,dy = 8x\frac{x^2}{2} = 4x^3,\ 0<x<1$ (6P)

$f_Y(y) = \int_y^1 8xy\,dx = 8y\left[\frac{x^2}{2}\right]_y^1 = 4y(1-y^2),\ 0<y<1$

(c) $E(X) = \int_0^1 4x^4\,dx = \underline{\frac{4}{5}}$, $E(Y) = \int_0^1 4y^2(1-y^2)\,dy = \underline{\frac{8}{15}}$, $E(X\cdot Y) = \int_0^1\int_0^x 8x^2y^2\,dy\,dx = \underline{\frac{4}{9}}$ (8P)

$Cov(X,Y) = E(X\cdot Y) - E(X)E(Y) = \frac{4}{9} - \frac{32}{75} = \underline{0.0178}$

(d) nein, wegen $Cov(X,Y) \neq 0$

⑤ $X_t = U\sin 2\pi t\cos 2\pi t = U\sin 4\pi t$, $U \sim U(0,1)$. $E(U) = \frac{1}{2}$, $E(U^2) = \frac{1}{3}$ (4P)

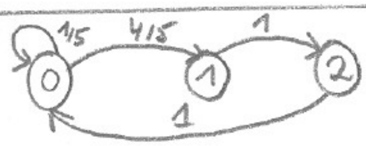
(a) $m_t = E(X_t) = E(U\cdot\sin 4\pi t) = \sin 4\pi t\, E(U) = \underline{\frac{1}{2}\sin 4\pi t}$

(b) $E(X_t\cdot X_s) = E(U^2\sin 4\pi t\sin 4\pi s) = \sin 4\pi t\sin 4\pi s\, E(U^2) = \underline{\frac{1}{3}\sin 4\pi t\sin 4\pi s}$ (6P)

(c) $K(s,t) = E(X_t\cdot X_s) - m_t\cdot m_s = \ldots = \underline{\frac{1}{12}\sin 4\pi t\sin 4\pi s}$ (8P)

(d) nein, da $K(s,t)$ Funktion von s und t. (2P)

⑥ (a) (4P)



(b) Grenzverteilung: $\underset{\sim}{p}(P-E) = \underset{\sim}{0}$ und $p_0+p_1+p_2 = 1$

$\underset{\sim}{p}(P-E) = (p_0\ p_1\ p_2)\begin{pmatrix} -4/5 & 4/5 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$:

$-4/5\,p_0 + p_2 = 0 \Rightarrow p_2 = \frac{4}{5}p_0$ (1)

$4/5\,p_0 - p_1 = 0$

$p_1 - p_2 = 0 \Rightarrow p_1 = p_2$ (3)

$p_0+p_1+p_2 = p_0 + \frac{4}{5}p_0 + \frac{4}{5}p_0 \overset{!}{=} 1 \Rightarrow \underline{p_0 = \frac{5}{13}} \overset{(1)}{\Rightarrow} \underline{p_2 = \frac{4}{13}},\ \underline{p_1 = \frac{4}{13}}$ (6P)

$p = \left(\frac{5}{13}, \frac{4}{13}, \frac{4}{13}\right)$

(c) $m_i = \frac{1}{p_i} \Rightarrow \underset{\sim}{m} = \left(\frac{13}{5}, \frac{13}{4}, \frac{13}{4}\right)$ (4P)

(d) $P^2 = P\cdot P = \begin{pmatrix} 1/25 & 4/25 & 20/25 \\ \boxed{1} & 0 & 0 \\ 1/5 & \boxed{4/5} & 0 \end{pmatrix}$, $P(X_4=0|X_2=1) = p^2_{10} = 1$, $P(X_4=1|X_2=2) = p^2_{21} = 4/5$ (6P)