LÖSUNGSBLATT WTH u. STOCH. PROZESSE
16.5.2003

① (a) $P(N_1) = \frac{|N_1|}{|\Omega|} = \frac{32}{128} = \underline{\underline{\frac{1}{4}}}$, $P(A_2) = \frac{50}{128} = \underline{0.3906}$, $P(A_3) = \frac{28}{128} = \frac{7}{32} = \underline{0.21875}$ (6P)

(b) $P(A_1|N_2) = \frac{|A_1 \cap N_2|}{|N_2|} = \frac{24}{64} = \underline{\frac{3}{8}}$, $P(N_2|A_1) = \frac{|A_1 \cap N_2|}{|A_1|} = \frac{24}{50} = \underline{0.48}$ (6P)

(c) $P(A_1|N_1 \cup N_2) = \frac{|A_1 \cap (N_1 \cup N_2)|}{|N_1 \cup N_2|} = \frac{44}{96} = \underline{0.458}$, $P(A_2|N_1 \cup N_2) = \frac{40}{96} = \underline{0.41\dot{6}}$,

$P(N_2 \cup N_3|A_1) = \frac{30}{50} = \underline{0.6}$ (8P)

② $N = 64$ Stück, $n = 4$, $M = 3$ defekt, $X_1 \sim H(64,4,3)$

$X_1 = 0$: Annahme, $X_1 = 1$: 2. Stichprobe, $X_1 \geq 2$: Ablehnung

(a) $P(2.\text{ Stichprobe}) = P_{X_1}(X_1 = 1) = \frac{\binom{3}{1}\binom{61}{3}}{\binom{64}{4}} = \underline{0.1699}$ (4P)

(b) $P(\text{Annahme}) = P(X_1 = 0) + P(X_1 = 1, X_2 = 0) = P(X_1 = 0) + P(X_1 = 1)\,P(X_2 = 0|X_1 = 1)$

$$= \underbrace{\frac{\binom{3}{0}\binom{61}{4}}{\binom{64}{4}}}_{\text{1. Stichprobe}} + \underbrace{\frac{\binom{3}{1}\binom{61}{3}}{\binom{64}{4}}}_{\text{1 Ausschuß}} \cdot \underbrace{\frac{\binom{2}{0}\binom{58}{8}}{\binom{60}{8}}}_{\substack{\text{2. Stichprobe kein Ausschuß} \\ N=60,\, n=8,\, M=2}} = 0.821 + 0.1699 \cdot 0.749 = \underline{0.94825}$$ (8P)

(c) $P(2\text{ Stichpr.}|\text{Ann.}) = \frac{P(2\text{ Stichpr. und Ann.})}{P(\text{Annahme})} = \frac{P(X_1 = 1, X_2 = 0)}{0.94825} = \frac{0.1272}{0.94825} = \underline{0.13414}$ (8P)

③ (a) $\int_0^1 f_X(x)\,dx = c\int_0^1 x^3(1-x)\,dx = \frac{c}{4} - \frac{c}{5} \stackrel{!}{=} 1 \Rightarrow \frac{c}{20} = 1 \Rightarrow \underline{c = 20}$ (4P)

(b) $F_X(x) = \int_0^x 20t^3(1-t)\,dt = \begin{cases} 0 & x < 0 \\ 5x^4 - 4x^5 = x^4(5-4x) & 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$ (4P)

(c) $E(X) = \int_0^1 x\,f_X(x)\,dx = \int_0^1 20x^4(1-x)\,dx = \frac{20}{5} - \frac{20}{6} = \frac{20}{30} = \underline{\frac{2}{3}}$

$E(X^2) = \int_0^1 x^2 f_X(x)\,dx = \int_0^1 20x^5(1-x)\,dx = \frac{20}{6} - \frac{20}{7} = \frac{20}{42} = \frac{10}{21}$ (8P)

$\text{Var}(X) = E(X^2) - E^2(X) = \frac{10}{21} - \frac{4}{9} = \underline{\frac{2}{63}} = \underline{0.0317}$

(d) $P_X(0.4 < X \leq 0.8) = F_X(0.8) - F_X(0.4) = 0.73728 - 0.08704$

$= \underline{0.65024}$ (4P)

④ (a) $E(G) = E(V+D) = E(V) + E(D) = 15 + 5 = 20$ [min] (4P)

$$\rho(V,D) = \frac{Cov(V,D)}{\sqrt{Var(V)\,Var(D)}} \Rightarrow Cov(V,D) = \rho(V,D)\sqrt{Var(V)\,Var(D)}$$
$$= -\frac{1}{2}\sqrt{8\cdot 2} = \underline{-2}$$

$$Cov(V,D) = E(V\cdot D) - E(V)E(D) \Rightarrow E(V\cdot D) = Cov(V,D) + E(V)E(D)$$
$$= -2 + 15\cdot 5 = \underline{73}$$

$$Var(G) = Var(V) + Var(D) + 2\,Cov(V,D) = 8 + 2 - 4 = \underline{\underline{6}}$$ (16P)

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⑤ $N_t \sim P(4\times t)$, $P(N_t = k) = \frac{(4t)^k}{k!}e^{-4t}$

(a) P(im Jahr höchstens 2 Störfälle) $= P(N_1 \le 2) = e^{-4}\left(1+4+\frac{4^2}{2}\right) = 13e^{-4} = \underline{0.2381}$ (4P)

(b) P(4. Störung nach 2 Jahren) $= P(T_4 > 2) = P(N_2 \le 3)$ (8P)
$$= \sum_{k=0}^{3}\frac{8^k}{k!}e^{-8} = e^{-8}\left(1+8+\frac{8^2}{2}+\frac{8^3}{6}\right) = \underline{0.04238}$$

(c) P(1. Jahr 2 Störungen | 3 Jahre mit 6 Störungen) $= P(N_1 = 2 \mid N_3 = 6)$
Formel (12.6) mit $u=1$, $t=3$, $k=2$, $n=6$: $= \binom{6}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4 = \underline{0.3292}$ (8P)

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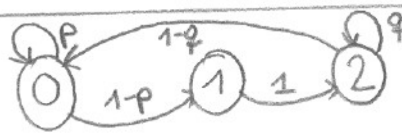
⑥ (a) (4P)



(b) Zustand 0 ist rekurrent, d.h. $f_0 = \sum_{n=1}^{\infty} f_0^{(n)} = 1$

Bew: $f_0^{(1)} = p$, $f_0^{(2)} = 0$, $f_0^{(3)} = (1-p)1(1-q)$, $f_0^{(n)} = (1-p)q^{n-3}(1-q)$, $n \ge 3$

$$f_0 = \sum_{n=1}^{\infty} f_0^{(n)} = p + (1-p)(1-q)\underbrace{\sum_{n=0}^{\infty} q^n}_{1/(1-q)} = p + (1-p) = \underline{1}$$ w.z.b.w. (8P)

(c)
$$m_0 = E(T_0) = \sum_{n=1}^{\infty} n f_0^{(n)} = p + (1-p)(1-q)\sum_{n=3}^{\infty} n q^{n-3}$$
$$= p + (1-p)(1-q)\left\{\sum_{k=0}^{\infty}(k+3)q^k\right\} = p + (1-p)(1-q)\left\{\sum_{k=1}^{\infty} k q^k + 3\sum_{k=0}^{\infty} q^k\right\}$$
$$= p + (1-p)(1-q)\left\{\frac{q}{(1-q)^2} + \frac{3}{1-q}\right\} = \frac{p(1-q) + (1-p)q + 3(1-p)(1-q)}{1-q}$$
$$= \frac{p - pq + q - pq + 3 - 3p - 3q + 3pq}{1-q} = \underline{\frac{3 - 2p - 2q + pq}{1-q}}$$ (8P)