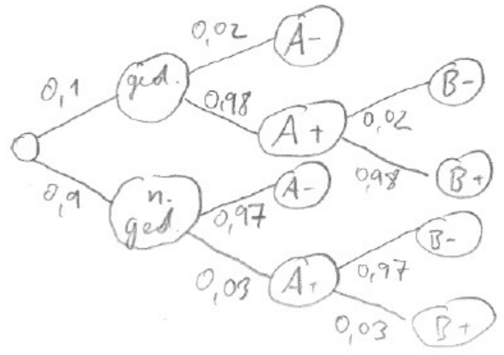
WTH II Stoch. Prozesse 11-03-2005

1 a)



b) $P(A+) =$

$P(A+ \cap ged) + P(A+ \cap n.ged) =$

$P(A+|ged) \cdot P(ged) + P(A+|n.ged) \cdot P(n.ged) =$

$0{,}98 \cdot 0{,}1 + 0{,}03 \cdot 0{,}9 = 0{,}125$

c) $P(A+ \cap B+ \mid n.ged) = 0{,}03^2 = 0{,}0009$

d) $$P(ged \mid A+ \cap B+) = \frac{P(A+ \cap B+ \mid ged) \cdot P(ged)}{P(A+ \cap B+ \mid ged) \cdot P(ged) + P(A+ \cap B+ \mid n.ged) \cdot P(n.ged.)}$$

$$= \frac{0{,}98^2 \cdot 0{,}1}{0{,}98^2 \cdot 0{,}1 + 0{,}03^2 \cdot 0{,}9} \approx 0{,}992$$

2 a) $$P(X=k) = \frac{\binom{32}{k} \cdot \binom{480}{16-k}}{\binom{512}{16}}$$

$\mathbb{E}X = 16 \cdot \frac{32}{512} = 1$

$\mathrm{Var}\,X = 16 \cdot \frac{32}{512} \cdot \left(1 - \frac{32}{512}\right) \cdot \frac{512-16}{511} = \frac{465}{511} \approx 0{,}91$

b) $$P(X=k) = \binom{16}{k} \cdot \left(\frac{1}{16}\right)^k \cdot \left(\frac{15}{16}\right)^{32-k}$$

$\left(\frac{1}{16} = \frac{32}{512}\right)$

$P_X(1 \leq X \leq 3) = 0{,}636$

c) (i) $P_X(1 \leq X \leq 3) \approx e^{-1}\left[1 + \frac{1}{2} + \frac{1}{6}\right] \approx 0{,}612$

(Hier kann auch die Varianz der Binomialverteilung stehen)

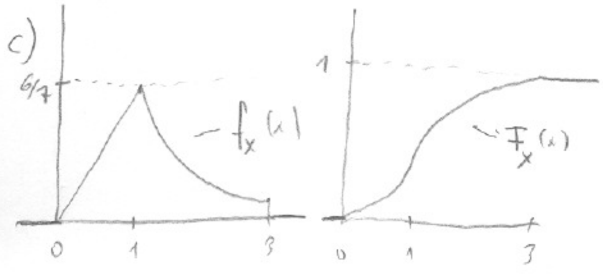
(ii) $P_X(1 \leq X \leq 3) \approx P(0{,}5 \leq N(1, \sqrt{0{,}91}) \leq 3{,}5) = P\left(\frac{-0{,}5}{\sqrt{0{,}91}} \leq N \leq \frac{2{,}5}{\sqrt{0{,}91}}\right) \approx 0{,}698$

3 a) $\int_0^1 cx\,dx + \int_1^3 \frac{c}{x^2}\,dx = 1 \iff c \cdot \frac{1}{2} + \left(-\frac{c}{3} + c\right) = 1 \iff c = \frac{6}{7}$

b) $$P(X \leq x) = \begin{cases} \frac{6}{7}\int_0^x t\,dt & (0 \leq x \leq 1) \\ \frac{6}{14} + \frac{6}{7}\int_1^x \frac{1}{t^2}\,dt & (1 < x \leq 3) \end{cases} \quad \text{also} \quad \begin{cases} \frac{3}{7}x^2 & (0 \leq x \leq 1) \\ \frac{3}{7} + \frac{6}{7}\left(1 - \frac{1}{x}\right) & (1 < x \leq 3) \end{cases}$$

$\left(\begin{array}{l} P(X \leq x) = 0 \quad (x < 0) \\ P(X \leq x) = 1 \quad (x \geq 3) \end{array}\right)$

c)



d) $P(1/2 \leq X \leq 3/2) = \frac{17}{28} = 0{,}607$

$\mathbb{E}X = \frac{6}{7}\left(\int_0^1 x^2\,dx + \int_1^3 x^{-1}\,dx\right) = \frac{6}{7}\left(\frac{1}{3} + \ln 3\right) = 1{,}23$

④ Papier: $\mu_p = 10$ $\quad \sigma_p^2 = 4$ $\qquad$ Karton $\mu_k = 30$ $\quad \sigma_k^2 = 36$

(a) $\text{Dicke} \sim N\left(185 \cdot 10 + 2 \cdot 30;\ \sqrt{185 \cdot 4 + 2 \cdot 36}\right) = N(1910;\ 28{,}5)$

$P(1800 \leq \text{Dicke} \leq 2000) = P\left(\frac{-110}{28{,}5} \leq N(0,1) \leq \frac{90}{28{,}5}\right) = \phi(3{,}16) - \phi(-3{,}86) \approx 1$

(b) $\text{Dicke } 185 \text{ Blätter} \sim N\left(1850;\ \sqrt{185 \cdot 4}\right) = N(1850;\ 27{,}2)$

$P(N(1850;\ 27{,}2) \leq 1500) = P\left(N(0,1) \leq \frac{1500 - 1850}{27{,}2}\right) = \phi(-12{,}87) = 0$

(c) $\text{Dicke von } k \text{ Skripten} \sim N(k \cdot 1910;\ \sqrt{k}\ 28{,}5)$

$P(N(k \cdot 1910;\ \sqrt{k}\ 28{,}5) > 24500) \leq 0{,}5 \iff$

$P\left(N(0,1) < \frac{24500 - k1910}{\sqrt{k}\ 28{,}5}\right) > 0{,}5 \iff \frac{24500 - k1910}{\sqrt{k}\ 28{,}5} > \phi^{-1}(0{,}5) = 0$

$\iff k < \frac{245\cancel{0}\cancel{0}}{191\cancel{0}}$ $\qquad$ also $k \leq 12$

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⑤ a) $\mathbb{E}X_n = a + \mathbb{E}Z_n + b\,\mathbb{E}Z_{n-1} + c\,\mathbb{E}Z_{n-2} = a$

$\operatorname{Var} X_n = \operatorname{Var} Z_n + b^2 \operatorname{Var} Z_{n-1} + c^2 \operatorname{Var} Z_{n-2} = \sigma^2 \cdot (1 + b^2 + c^2)$

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b) $K(n, n+1) = \mathbb{E}X_n X_{n+1} - \mathbb{E}X_n \mathbb{E}X_{n+1} \underset{\substack{\uparrow \\ \mathbb{E}Z_k Z_l = 0 \text{ für } k \neq l}}{=} \cancel{a^2} + \sigma^2 b + cb\sigma^2 - \cancel{a^2} = \sigma^2 b(1+c)$

genauso zeigt man $K(n, n+2) = \mathbb{E}X_n X_{n+2} - \mathbb{E}X_n \mathbb{E}X_{n+2} = c\sigma^2$

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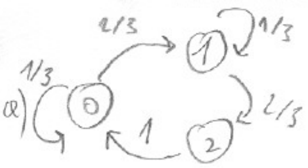
⑥ a)



b) $(p_0, p_1, p_2) \cdot \begin{pmatrix} 1/3 & 2/3 & 0 \\ 0 & 1/3 & 2/3 \\ 1 & 0 & 0 \end{pmatrix} = \left(p_0/3 + p_2,\ \tfrac{2}{3}p_0 + \tfrac{1}{3}p_1,\ \tfrac{2}{3}p_1\right)$

Löse: $(p_0, p_1, p_2) = \left(p_0/3 + p_2,\ \tfrac{2}{3}p_0 + \tfrac{1}{3}p_1,\ \tfrac{2}{3}p_1\right)$ mit

Nebenbedingung $p_0 + p_1 + p_2 = 1 \Rightarrow p = (3/8,\ 3/8,\ 1/4)$

c) $m = \left(\frac{8}{3}, \frac{8}{3}, 4\right)$

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d) $\left.\begin{array}{l} P(X_3 = 1 \mid X_1 = 0) = \frac{1}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{1}{3} = \frac{4}{9} \\ P(X_3 = 0 \mid X_1 = 2) = \frac{1}{3} \end{array}\right\}$ durch Hinschauen auf a)