LÖSUNGSBLATT WTH u. STOCH. PROZESSE

12.3.2004

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① 14 Kugeln, 5 rot, 5 grün, 4 blau

3 mal ziehen (i) ohne Zurücklegen, (ii) mit Zurücklegen

(a) $P(\text{von jeder Farbe eine Kugel}) = P_{rgb} \times 3!$

(i) $\frac{5\cdot5\cdot4}{14\cdot13\cdot12}\cdot3! = \frac{25}{91} = \underline{0.2747}$, (ii) $\frac{5\cdot5\cdot4}{14\cdot14\cdot14}\cdot3! = \frac{75}{343} = \underline{0.2187}$ (6P)

(b) $P(\text{3 Kugeln mit gleicher Farbe}) = P_{rrr} + P_{ggg} + P_{bbb}$

(i) $\frac{5\cdot4\cdot3+5\cdot4\cdot3+4\cdot3\cdot2}{14\cdot13\cdot12} = \frac{18}{271} = \underline{0.0658}$, (ii) $\frac{5^3+5^3+4^3}{14^3} = \frac{157}{1372} = \underline{0{,}1144}$ (6P)

(c) $P(\text{zuerst 2 rote Kugeln und dann \underline{keine} blaue}) = P_{rrg} + P_{rrr}$

(i) $\frac{5\cdot4\cdot5+5\cdot4\cdot3}{14\cdot13\cdot12} = \frac{165}{2184} = \underline{0.07326}$ (ii) $\frac{5^3+5^3}{14^3} = \frac{125}{1372} = \underline{0.091}$ (4P)

(d) $P(\text{2. und 3. Zug gleiche Farbe} \mid \text{1. Zug blau}) = P_{rr|b} + P_{gg|b} + P_{bb|b}$

(i) $\frac{5\cdot4+5\cdot4+4\cdot3}{13\cdot12} = \frac{23}{78} = \underline{0.2949}$, (ii) $\frac{5^2+5^2+4^2}{14^2} = \frac{33}{98} = \underline{0{,}3367}$ (4P)

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② (a) $X \sim B(150, 0.02)$, $E(X) = 150\cdot0{,}02 = 3$, $Var(X) = 2.94$ (8P)

$$P_X(X \le 2) = \sum_{k=0}^{2}\binom{150}{k}0{,}02^k\cdot0{,}98^{150-k} = \underline{0.421}$$

(b) $X \sim B(1000, 0.02)$, approx: $X \sim P(20)$, $X \sim N(20, \sqrt{20\cdot0.98})$

(i) $P_X(18 \le X \le 22) \approx \sum_{k=18}^{22}\frac{20^k}{k!}e^{-20} = e^{-20}\cdot\frac{20^{18}}{18!}(\ldots) = \underline{0.424}$

(ii) $P_X(18 \le X \le 22) \approx \Phi\Big(\underbrace{\frac{22-20+1/2}{\sqrt{20\cdot0.98}}}_{x=0.5647}\Big) - \Phi(-x) = 2\Phi(x) - 1 = \underline{0.4277}$ (12P)

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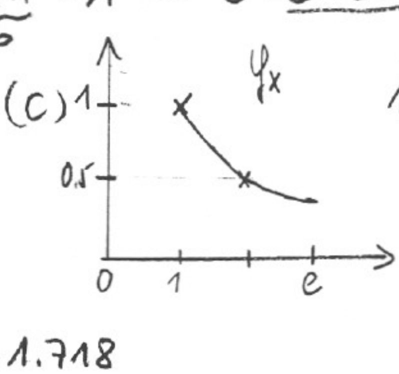
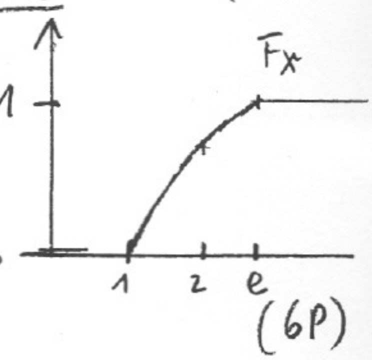
③ (a) $\int_1^b \frac{1}{x}dx = \ln x\Big|_1^b = \ln b - \underbrace{\ln 1}_{=0} \overset{!}{=} 1 \Rightarrow b = \underline{e^1 = 2.718}$ (4P)

(b) $F_X(x) = \begin{cases} 0 & x < 1 \\ \ln x & 1 \le x \le e \\ 1 & e < x \end{cases}$ (4P)

(c)





(6P)

(d) $E(X) = \int_1^e \frac{1}{x}x\,dx = \int_1^e dx = e - 1 = 1.718$ (6P)

$F_X(x_{0.5}) = \frac{1}{2} \Leftrightarrow \ln x_{0.5} = \frac{1}{2} \Rightarrow x_{0.5} = e^{1/2} = 1{,}6487 < E(X)$, ja. (6P)

④ $D \sim N(40,8)$, $J \sim N(90,16)$, $F \sim N(100,20)$

(a) $W = D + J + F \sim N(40+90+100, \sqrt{8^2+16^2+20^2}) = N(230, \underbrace{\sqrt{480}}_{26.83})$

(4P)

(b) $\bar{X} = \frac{1}{5}\sum_{i=1}^{5}(D_i + J_i) \sim N\left(40+90, \sqrt{\frac{64+256}{5}}\right) = N(130, 8)$ (8P)

(c) $D = J - F \sim N(90-100, \sqrt{256+400}) = N(-10, \underbrace{\sqrt{656}}_{25.6125})$

$P_D(D>0) = 1 - P_D(D \leq 0) = 1 - \Phi\left(\frac{0-(-10)}{25.6125}\right) = 1 - \Phi(0.39) = \underline{0.348}$ (8P)

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⑤ $X_t = A \cos \omega t$, $A \geq 0$ ZV mit $E(A) = \mu$, $Var(A) = \sigma$

(a) $m_t = E(X_t) = E(A \cdot \cos \omega t) = \cos \omega t \, E(A) = \mu \cos \omega t$ (6P)

$\sigma_t^2 = Var(X_t) = Var(A \cdot \cos \omega t) = \cos^2 \omega t \, Var(A) = \sigma \cos \omega t$

(b) $K(s,t) = E(X_s \cdot X_t) - m_t \cdot m_s = E(A^2 \cdot \cos \omega t \cdot \cos \omega s) - \mu^2 \cos \omega t \cos \omega s$ (8P)

$= \cos \omega t \cdot \cos \omega s \, (E(A^2) - \mu^2) = Var(A) \cdot \cos \omega t \cos \omega s$ (4P)

(c) $\rho(s,t) = \frac{K(s,t)}{\sqrt{Var(X_t) Var(X_s)}} = \frac{Var(A) \cdot \cos \omega t \cos \omega s}{Var(A) \cos \omega t \cos \omega s} = 1$ (2P)

d) nein, da m_t Funktion von t

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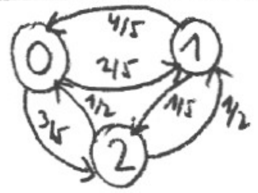
⑥ (a)



(4P)

(b) $p^{(1)} = p^{(0)} \cdot P = \left(\frac{2}{5} \; \frac{2}{5} \; \frac{1}{5}\right) \begin{pmatrix} 0 & 2/5 & 3/5 \\ 4/5 & 0 & 1/5 \\ 1/2 & 1/2 & 0 \end{pmatrix} = \left(\frac{21}{50} \; \frac{13}{50} \; \frac{16}{50}\right)$ (6P)

(c) Grenzverteilung $p(P-E) = \underline{0}$ mit $p_0 + p_1 + p_2 = 1$

$p(P-E) = (p_0 \; p_1 \; p_2) \begin{pmatrix} -1 & 2/5 & 3/5 \\ 4/5 & -1 & 1/5 \\ 1/2 & 1/2 & -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$! $-p_0 + \frac{4}{5}p_1 + \frac{1}{2}p_2 = 0$

$\frac{2}{5}p_0 - p_1 + \frac{1}{2}p_2 = 0$

3. Zl linear abh.

1. Zeile ergibt $p_0 = \frac{4}{5}p_1 + \frac{1}{2}p_2$, eingesetzt in 2. Zeile liefert $p_2 = \frac{34}{35}p_1$, substituiert in $p_0 + p_1 + p_2 = 1$ ergibt $\frac{9}{5}p_1 + \frac{51}{35}p_1 = 1 \Rightarrow p_1 = \frac{35}{114} \Rightarrow p_2 = \frac{34}{114} \Rightarrow p_0 = 1 - p_1 - p_2 = \frac{45}{114}$.

$p = \left(\frac{45}{114}, \frac{35}{114}, \frac{34}{114}\right) = (0.3974, 0.3070, 0.29825)$ (10P)