LÖSUNGSBLATT WTH u. STOCH. PROZESSE

31. 1. 2003

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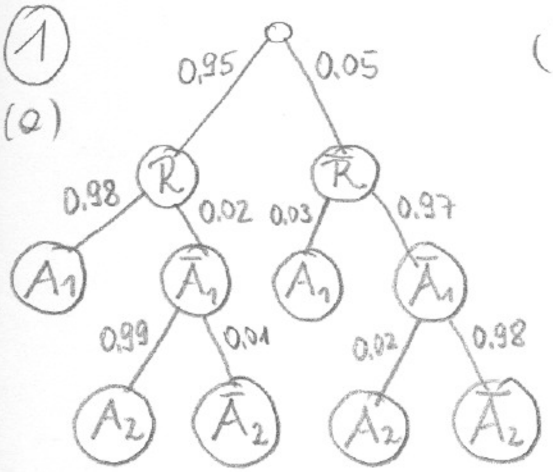
(1)

(a)



(6P)

(b) P(richtig eingeordnet)

$= P(R\cap A_1) + P(R\cap\overline{A}_1\cap A_2) + P(\overline{R}\cap\overline{A}_1\cap\overline{A}_2)$

$= 0.95\times 0.98 + 0.95\times 0.02\times 0.99 + 0.05\times 0.97\times 0.98$

$= \underline{0.99734}$ (6P)

(c) $P(\text{registr.}\mid\text{abgewiesen}) = P(E\mid\overline{A}_2) = \dfrac{P(E\cap\overline{A}_2)}{P(\overline{A}_2)} = \dfrac{0.95\times 0.02\times 0.01}{0.95\times 0.02\times 0.01 + 0.05\times 0.97\times 0.98}$

$= \dfrac{0.00019}{0.04772} = \underline{0.00398}$

(8P)

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(2) (a) $X \sim B(12, 0.6)$: $P_X(X=k) = \binom{12}{k} 0.6^k \cdot 0.4^{12-k}$, $k = 0, \dots, 12$

$E(X) = n\cdot p = 12\times 0.6 = 7.2$, $Var(X) = npq = 12\times 0.6\times 0.4 = 2.88$ (6P)

(b) $P_X(X\le 5) = \sum_{k=0}^{5} \binom{12}{k} 0.6^k \cdot 0.4^{12-k} = \underline{0.1582}$, $\Phi\left(\frac{5.5-7.2}{\sqrt{2.88}}\right) = \Phi(-1) = \underline{0.159}$ [Appr.] (4P)

(c) $P_X(6\le X\le 9) = \sum_{k=6}^{9}$ — " — $= \underline{0.7583}$, $\Phi\left(\frac{9.5-7.2}{\sqrt{2.88}}\right) - \Phi\left(\frac{5.5-7.2}{\sqrt{2.88}}\right) = \underline{0.753}$ (4P)

(d) $Y \sim B(12, 0.9)$: $P_Y(Y\ge 10) = \sum_{k=10}^{12} \binom{12}{k} 0.9^k \times 0.1^{12-k} = 0.889 > 0.85$, $\underline{ja}$ (6P)

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(3) (a) z.z. $\int_{\mathbb{R}} f_X(x)\,dx = 1$: $\int_0^a \frac{1}{2a}\,dx + \int_a^\infty \frac{1}{2a} e^{-(x-a)/a}\,dx = \frac{1}{2} + \frac{1}{2a}\underbrace{\left[-a e^{-(x-a)/a}\right]_a^\infty}_{0+a} = \underline{1}$ (6P)

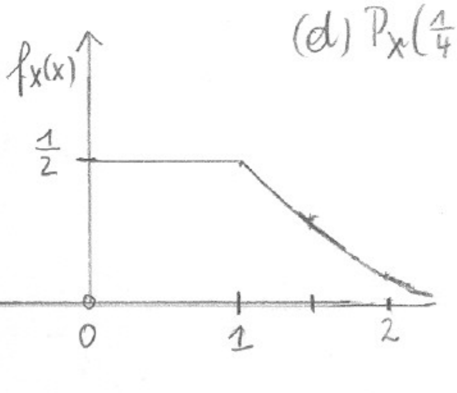
(b) $F_X(x) = \begin{cases} 0 & x<0 \\ \int_0^x \frac{1}{2a}\,dt = x/2a & 0\le x\le a \\ \int_0^a \frac{1}{2a}\,dt + \int_a^x \frac{1}{2a} e^{-(t-a)/a}\,dt = \frac{1}{2} + \frac{1}{2a}\left[-a e^{-(t-a)/a}\right]_a^x = \frac{1}{2} - \frac{1}{2} e^{-(x-a)/a} + \frac{1}{2} & a<x<\infty \end{cases}$

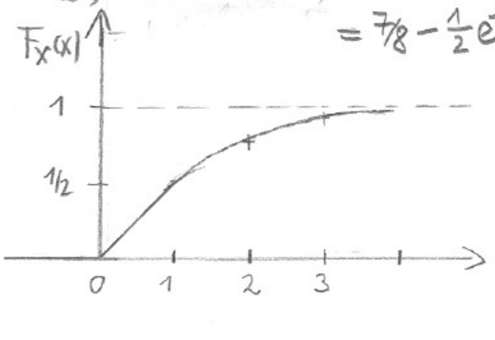
(4P)

(d) $P_X\left(\frac{1}{4} < X \le \frac{3}{2}\right) = F_X\left(\frac{3}{2}\right) - F_X\left(\frac{1}{4}\right) = 1 - \frac{1}{2}e^{-1/2} - \frac{1}{8}$

$= 7/8 - \frac{1}{2}e^{-1/2} = \underline{0.5717}$ (6P)

(c) $\underline{a=1}$





(4P)

4

(a)

$X \backslash Y$	0	1	2	$P_Y(Y=j)$
0	1/3	1/12	1/12	1/2
1	0	1/12	1/12	1/6
2	1/6	0	1/6	1/3
$P_X(X=i)$	1/2	1/6	1/3	1

(4P)

(b)

$E(X)$	$0 + \frac{1}{6} + \frac{2}{3} = \frac{5}{6} = E(Y)$ (X,Y haben dieselbe Verteilung!)
$E(X^2)$	$0 + \frac{1}{6} + \frac{4}{3} = \frac{3}{2} = E(Y^2)$
$Var(X)$	$E(X^2) - E^2(X) = \frac{3}{2} - \frac{25}{36} = \frac{29}{36} = Var(Y)$

$E(X \cdot Y) = 1 \cdot 1 \cdot \frac{1}{12} + 1 \cdot 2 \cdot \frac{1}{12} + 2 \cdot 2 \cdot \frac{1}{6} = \underline{\frac{11}{12}}$, $Cov(X,Y) = \frac{11}{12} - \left(\frac{5}{6}\right)^2 = \underline{\underline{\frac{2}{9}}}$

$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}} = \frac{2/9}{29/36} = \underline{\frac{8}{29}}$ (8P)

(c) $\underline{Z = X+Y}$: $P_Z(Z=k) = \sum_{i=0}^{k} P_{X,Y}(X=i, Y=k-i)$:

k	0	1	2	3	4	Σ
$P_Z(Z=k)$	1/3	1/12	1/3	1/12	1/6	1

(8P)

5

$N_t \sim P(\frac{1}{10} \times t)$ (4P)

(a) $P(N_4 \leq 1) = e^{-0.4} + 0.4e^{-0.4} = \underline{0.9384}$

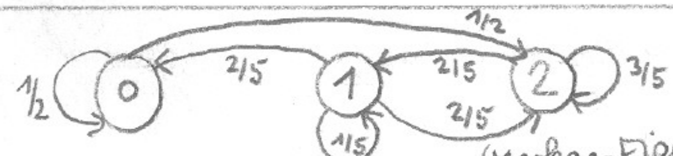
(b) $P(N_6 - N_4 = 0, N_{12} - N_6 = 1) \overset{unabh.}{=} P(N_2 = 0) \cdot P(N_6 = 1) = e^{-0.2} \times 0.6e^{-0.6} = \underline{0.2696}$ (6P)

(c) $P(N_3 - N_2 = 0 \mid N_2 - N_0 = 0) \overset{unabh. ZW}{=} P(N_3 - N_2 = 0) = P(N_1 = 0) = e^{-0.1} = \underline{0.9048}$ (4P)

(d) $\{T_2 > 6\} \Leftrightarrow \{N_6 \leq 1\}$: $P(T_2 > 6) = P(N_6 \leq 1) = e^{-0.6} + 0.6e^{-0.6} = \underline{0.8781}$ (6P)

6

(a)



(4P)

(b) $P(X_2 = 2 \mid X_1 = 0, X_0 = 1) \overset{Def. (Markov-Eigenschaft)}{=} P(X_2 = 2 \mid X_1 = 0) = p_{02} = \underline{\underline{\frac{1}{2}}}$

$P(X_2 = 2, X_1 = 0 \mid X_0 = 1) = P(X_2 = 2 \mid X_1 = 0, X_0 = 1)\, P(X_1 = 0 \mid X_0 = 1)$

$\overset{Def.}{=} P(X_2 = 2 \mid X_1 = 0)\, P(X_1 = 0 \mid X_0 = 1) = p_{02} \times p_{10} = \frac{1}{2} \cdot \frac{2}{5} = \underline{\underline{\frac{1}{5}}}$

(6P)

(c) $P(X_2 = 2, X_1 = 0 \mid X_0 = 0) = P(X_2 = 2 \mid X_1 = 0)\, P(X_1 = 0 \mid X_0 = 0)$

$= P(X_{n+1} = 2 \mid X_n = 0)\, P(X_n = 0 \mid X_{n-1} = 0) = \frac{1}{2} \cdot \frac{1}{2} = \underline{\underline{\frac{1}{4}}}$

$p^{(0)} = \left(\frac{2}{5}, \frac{3}{10}, \frac{3}{10}\right)$; $p^{(1)} = p^{(0)} \cdot P = \left(\frac{8}{25}, \frac{9}{50}, \frac{1}{2}\right)$

$P(X_1 = 2) = p_2^{(1)} = \underline{\underline{\frac{1}{2}}}$, $P(X_1 = 2, X_2 = 1) = P(X_2 = 1 \mid X_1 = 2)\, P(X_1 = 2)$

$= p_{21} \times p_2^{(1)} = \frac{2}{5} \cdot \frac{1}{2} = \underline{\underline{\frac{1}{5}}}$ (10P)