A Consideration of Error Probability in Multilevel Reliability Verification

Oesterreichische Statistiktage, Graz

Nikolaus Haselgruber

CIS - Consulting in Industrial Statistics

September 09, 2011

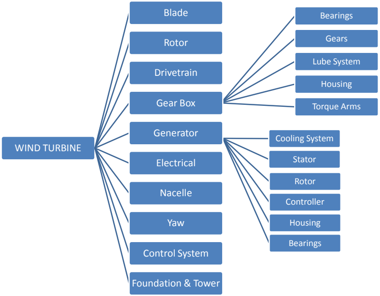
What are adequate sample sizes for component reliability verification tests?

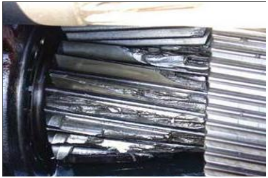
Reliability is an item's capability to

- provide its intended function
- over a specified time interval
- under defined usage conditions*

*compare, e.g., Meeker & Escobar [1998]

A Wind Turbine (Extract, Simplified)



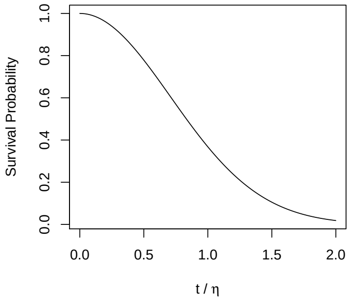


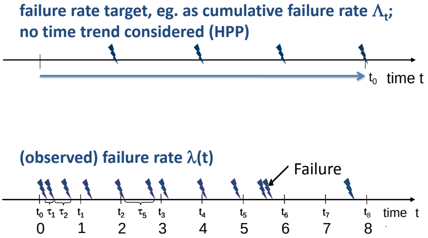
- The wind turbine is a repairable system, any failure will be removed by replacement of one or several components
- A failed component is non-repairable and has to be replaced

Reliability Verification Testing

- Scope of test object increases during development process
  - Component $\rightarrow$ subsystem $\rightarrow$ system
  - Comparability of tests on different levels is limited
- Apply sampling procedure iteratively on each relevant hierarchy level
- Adequate reliability targets required to determine required test samples (see Haselgruber [2011])

2 Reliability Models





Non-repairable (component)

- Reliability is a survival probability $R(t) = P(t > t_0)$
- $0 \leq R(t) \leq 1$
- Model: lifetime distribution

Repairable (system)

- Reliability is a failure intensity rate $\lambda(t)$
- $0 \leq \lambda(t) < \infty$
- Model: point process

Characteristics of the Reliability Models (Rigdon [2000])

Hazard rate of a lifetime distribution (component)

$$h(t_0) = \lim_{\epsilon \downarrow 0} \frac{1}{\epsilon} P(t_0 < t \leq t_0 + \epsilon | t > t_0) = -\frac{dR(t)}{R(t)dt}\bigg|_{t_0} = -\frac{d \ln R(t)}{dt}\bigg|_{t_0}$$

Cumulative hazard rate: $H(t_0) = \int_0^{t_0} h(u)du = -\ln R(t_0)$
$\Rightarrow$ based on conditional probability for occurrence of **the** failure

Failure rate of a point process (system) with counting variable $N(t)$ and mean function $\Lambda(t) = E(N(t))$

$$\lambda(t_0) = \lim_{\epsilon \downarrow 0} \frac{1}{\epsilon} P(N(t_0 < t \leq t_0 + \epsilon) \geq 1) = \frac{d\Lambda(t)}{dt}$$

$\Rightarrow$ based on unconditional probability for occurrence of **a** failure

Reliability Target

- Reliability target is given for a complete system, typically as cumulative failure rate $\Lambda_{t_0} = \Lambda(t_0) = \int_0^{t_0} \lambda dt = \lambda t_0$
- Usually no time trend is considered $\rightarrow \lambda = \text{const.}$
- To be decomposed recursively into component targets $R_{t,1}(t_0), R_{t,2}(t_0), \ldots$
- System structure and model characteristics to be considered

System Model for Reliability Target Allocation

- Homogeneous Poisson Process (HPP) with mean Λ_{t_0} as series system reliability model for decomposition
- $\Lambda_{t_0} = \sum_{j=1}^{m} \Lambda_{t_0,j}$ is decomposed in m component targets
- Component reliability target results in $R_{t,j}(t_0) = \exp(-\Lambda_{t_0,j})$ if $\Lambda_{t_0,j} = H_j(t_0)$
- Use risk weights $w_j = f$(development/technology risk, warranty experience, repair costs) for decomposition: $\Lambda_{t_0,j} = w_j * \Lambda_{t_0} / \sum_{i=1}^{m} w_i$

Component Model for Reliability Demonstration

Verification check for component $j, j = 1, \ldots, m$

- Component target $R_{t,j}(t_0)$ is given
- Weibull distributed lifetime $t_j \sim \mathbf{Wb}(\eta_j, \beta_j)$, shape β_j is known
- Success run sample with (right censored) durations $t_{j1}, \ldots, t_{jn_j}$
- Estimate demonstrable reliability

$$\underline{R_j}(t_0|\beta_j, \mathbf{t}_j = (t_{j1}, \ldots, t_{jn_j})) = \exp(-(\frac{t}{\underline{\eta_j}})^{\beta_j})$$

- Compare $\underline{R_j}(t_0)$ with $R_{t,j}(t_0)$

Parameter Estimation (compare Haselgruber [2007])

Likelihood function $L(\eta_j|\beta_j, \mathbf{t}_j)$ for success runs

$$L(\eta_j| \ldots) = \prod_{i=1}^{n_j} \exp\left(-\left(\frac{t_{ji}}{\eta_j}\right)^{\beta_j}\right) = \exp\left(-\eta_j^{-\beta_j} \sum_{i=1}^{n_j} t_{ji}^{\beta_j}\right)$$

Point estimation $\hat{\eta}_j$ for η_j from

$$\frac{\partial \ln L(\eta_j|\beta_j, \mathbf{t}_j)}{\partial \eta_j} = \frac{\partial l(\eta_j|\beta_j, \mathbf{t}_j)}{\partial \eta_j} = \frac{\beta_j}{\eta_j^{\beta_j+1}} \sum_{i=1}^{n_j} t_{ji}^{\beta_j} \overset{\eta_j \to \infty}{\longrightarrow} 0.$$

Conservative α-confidence bound for η_j based on $\hat{\eta}_j^{-\beta_j} \sim \mathbf{Ex}(1/\sum_{i=1}^{n_j} t_{ji}^{\beta_j})$ is

$$\underline{\eta_j} = (\sum_{i=1}^{n_j} t_{ji}^{\beta_j}/(-\ln \alpha))^{1/\beta_j}$$

Composite Components to a System

Classical situation for m series components:

- $P(R_1(t_0) > R_{t,1}(t_0), \ldots, R_m(t_0) > R_{t,m}(t_0)) = 1 - \alpha$
- $P(\prod_{j=1}^{m} R_j(t_0) > R_t(t_0)) = 1 - \alpha$
- $P(\sum_j \ln R_j(t_0) > \ln R_t(t_0)) = 1 - \alpha$

Distribution of $\ln R(t_0)$

- For each component it is $t_j \sim \mathbf{Wb}(\eta_j, \beta_j)$ with known β_j
- Thus, $\ln R_j(t_0) = -(t_0/\eta_j)^{\beta_j}$
- From (1) we know $\hat{\eta}_j^{-1/\beta_j} \sim \mathbf{Ex}(1/\sum_{i=1}^{n_j} t_{ji}^{\beta_j})$
- Consequently, $-\ln R_j(t_0) \sim \mathbf{Ex}(t_0^{\beta_j}/\sum_{i=1}^{n_j} t_{ji}^{\beta_j})$
- And, with the Exponential addition theorem,

$$-\ln R(t_0) = -\sum_j \ln R_j(t_0) \sim \mathbf{Ex}(\lambda_0),$$

  with $\lambda_0 = \sum_{j=1}^{m} \left(t_0^{\beta_j} / \sum_{i=1}^{n_j} t_{ji}^{\beta_j} \right)$.
- So, $P(-\ln R(t_0) < -\ln R_t(t_0)) = 1 - R_t(t_0)^{\lambda_0}$

Consequences of the Verification Check

- Targets $R_{t,j}(t_0)$ are derived from HPP
- In general, super-imposition of Weibull distributed time between failures does NOT lead to an HPP

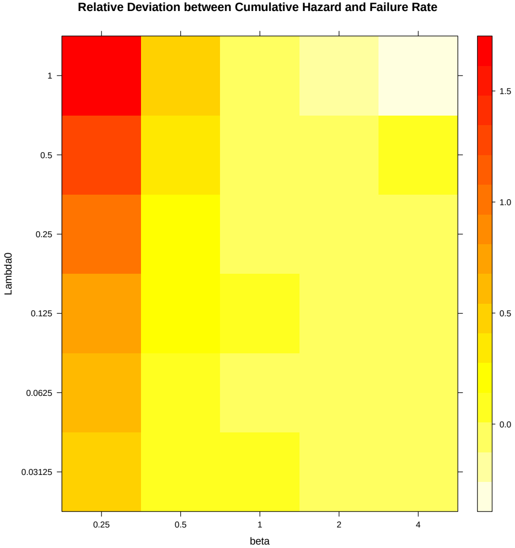
$\Rightarrow$ Comparison of $R_{t,j}(t_0)$ and $R_t(t_0)$ is restricted to the point t_0

$\Rightarrow$ Simulation shows good accurracy for tight targets $\Lambda_{t_0,j} \approx 0$ (relevant case)

Deviation between Cumulative Hazard and Failure Rate

- Simulation of deviation between cumulative hazard $H(t = t_0)$ and cumulative failure rate Λ_{t_0} for $t \sim \mathbf{Wb}(\eta, \beta)$
- Parameters of computer simulation experiment
  - $\beta \in 2^{\{-2,\ldots,2\}}$
  - $\Lambda_{t_0} \in 2^{\{-5,\ldots,0\}}$
- Because of $E(t) = \eta\Gamma(1 + 1/\beta)$, η was set to $t_0/(\Lambda_{t_0} * \Gamma(1 + 1/\beta))$

Relative Deviations



- Consider $\delta(t_0) = (H(t_0) - \Lambda_{t_0})/H(t_0)$
- $\Lambda_{t_0} \downarrow \Rightarrow \delta(t_0) \to 0$
- $\beta < 1$ & $\Lambda_{t_0} \uparrow \Rightarrow \delta(t_0) \uparrow$
- Relevant cases for component reliability verification: $\Lambda_{t_0} \downarrow$ & $\beta > 1$

Sampling for Verification

What are adequate samples $\mathbf{t}_j = (t_{1j}, \ldots, t_{n_j,j}), j = 1, \ldots, m$, to fulfill

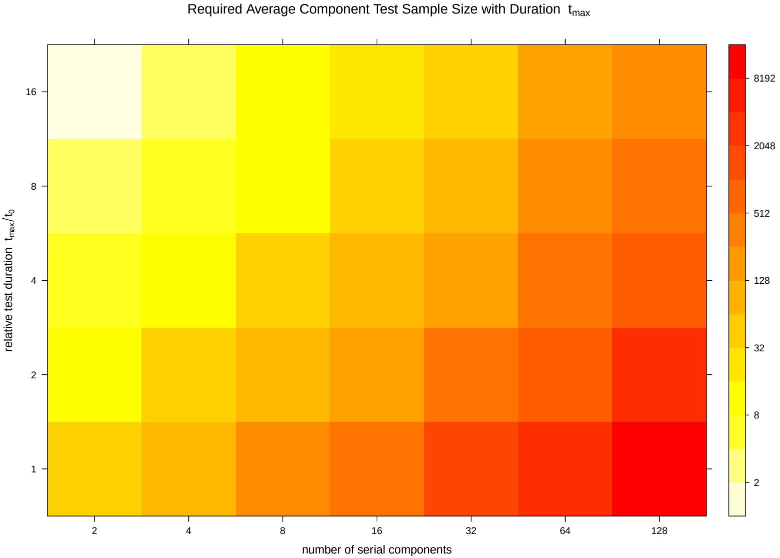
$$P(R(t_0) > R_t(t_0)) > 1 - \alpha?$$

Let all (planned) t_{ij} be equal so that we search for adequate n_j.

Simulation Parameters for Sampling

- $t_0 = 1$
- $\Lambda_{t_0} = 0.1$ (per t_0)
- $\beta_j \sim \mathbf{Wb}(\eta_\beta = 2.5; \beta_\beta = 5)$
- number of system's components $\in 2^{\{1,\ldots,7\}}$
- $t_{\max} = \{1, 2, 4\} * t_0$
- $t_{ij} = t_{\max} \quad \forall i, j$
- $n_j \geq 1 \quad \forall i, j$
- $\alpha = 0.1$

Sample Size Map for $P(\Lambda(t_0) < 0.1) > 0.9$



Consequences of the Success Run Sampling

Success run sample for minimum effort verification program

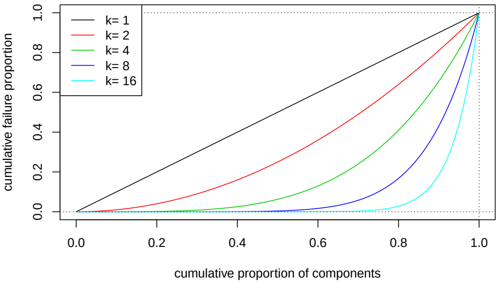
- Test results don't allow reliability point estimation

$\Rightarrow$ either conservative lower confidence bound but (in general still too) high test effort

$\Rightarrow$ or consider technical prior information

Present and Future Work

- Failure concentration
  - Failure frequency typically is concentrated on a few problematic components
  - Demonstrate for each component that it is not the one causing the top failure frequency
  - Use Gini coefficient for modeling the failure concentration
  - $\Rightarrow$ Relaxation of the targets
- Assume a prior distribution to model the uncertainty of β_j



Summary

- Overlength helps to decrease the total test time
- Cumulative hazard rate / failure tends to be
  - > 0 basically for small β, increasing with Λ_{t_0}
  - < 0 for larger β only if $\Lambda_{t_0} \geq 1$
  - ≈ 0 else, i.e., for the practically relevant cases

References and Acknowledgements

References

- Meeker, W., L. Escobar [1998]: Statistical Methods for Reliability Data. Wiley, New York.
- Rigdon, S. E., A. P. Basu [2000]: Statistical Methods for the Reliability of Repairable Systems. Wiley, New York.
- Singpurwalla, N.D. [2006]: Reliability and Risk. A Bayesian Perspective. Wiley, Chichester.
- Haselgruber, N. [2007]: Sampling and Design of Large-Scale Life Time Experiments. PhD Dissertation. University of Technology, Graz.
- Haselgruber, N., et.al. [2011]: Identification of adequate reliability targets. Proceedings of the 11th ENBIS Conference. Coimbra, Portugal.

Acknowledgements

- Science Park Graz
- FFG