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Female Wage Careers - A Bayesian Analysis Using Markov Chain Clustering

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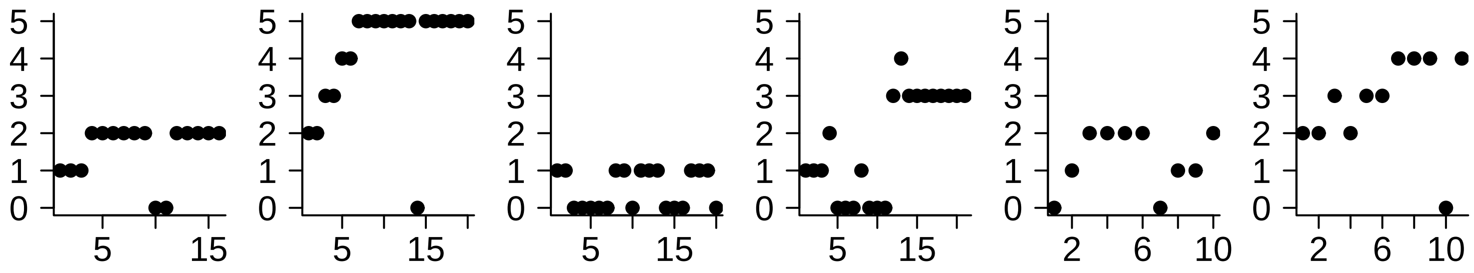
Outline

1. Analyzing wage dynamics
2. The data
3. The method:
   - Markov chain clustering
   - Mixture-of-experts Model
   - MCMC
4. Results
5. Conclusions

Analyzing Wage Dynamics

We analyze female wages **over a time period**.

6 income categories: 0 = no income, 1-5 quintiles of the income distribution



Q1 Are there **groups** of women with similar patterns in their wage dynamic?

Q2 Which **variables** influence the wage dynamic?

The Data

- Austrian Security Data Base with 183 805 female employees
- entry in the labor market between 1980 and 1985
- observation period till 2001 (in 2002 change of qualifying conditions for maternity leave)
- time series length: up to 22 years (median length: 14 years)
- adjusted for long-term unemployed (ts cut after five years of zero-income)

The Data

- age at entry: 14 to 25 years (64.2 % were 17-19 years old)
- start as blue collar worker: 41.1 %; as white collar worker: 58.9 %
- at least once on maternity leave: 72.7 %
- number of children: number of live birth announcements ($\bar{x} = 1.42$, $\tilde{x}_{0.5} = 1$)

see Zweimüller et al. (2009)

Model Based Clustering

$\mathbf{y}_i = \{y_{i1}, \ldots, y_{i,T_i}\}$ ... time series of income states for individual i
$y_{it} \in \{0, \ldots, K\}$ for $i = 1, \ldots, N, t = 1, \ldots, T_i$

Finite mixture model with H components:

$$\sum_{h=1}^{H} \eta_h \, p(\mathbf{y}_i | \boldsymbol{\xi}_h)$$

$\boldsymbol{\xi}_h$ describes the time-series of group h
η_h ... group specific weights

see Frühwirth-Schnatter & Kaufmann (2008)

Markov Chain Model

First-order time-homogeneous Markov chain model:

$$\xi_{jk} = \Pr(y_{it} = k | y_{i,t-1} = j) \qquad \text{and} \qquad \sum_{k=0}^{K} \xi_{jk} = 1$$

$$\boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{\xi}_{0\cdot} \\ \boldsymbol{\xi}_{1\cdot} \\ \vdots \\ \boldsymbol{\xi}_{K\cdot} \end{pmatrix} = \begin{pmatrix} \xi_{00} & \xi_{01} & \cdots & \xi_{0K} \\ \xi_{10} & \xi_{11} & \cdots & \xi_{1K} \\ \vdots & & \ddots & \vdots \\ \xi_{K0} & \xi_{K1} & \cdots & \xi_{KK} \end{pmatrix}$$

Each row represents an unknown discrete probability distribution.

Markov Chain Model

We introduce Markov chain models as clustering kernels. All time series within a cluster are described by the same cluster-specific transition matrix $\boldsymbol{\xi}_h$.

$$p(\mathbf{y}_i|\boldsymbol{\xi}_h) = \prod_{j=0}^{K} \prod_{k=0}^{K} (\xi_{h,jk})^{N_{i,jk}}$$

$N_{i,jk} = \#\{y_{it} = k, y_{i,t-1} = j\}$ is the number of switches of individual i from state j to state k

see Frühwirth-Schnatter & Pamminger (2010)

Modeling Group Membership

We incorporate unit-specific information to assign each individual to one group.

Multinomial Logit Model (MNL):

$$\Pr(S_i = h | \mathbf{x}_i, \boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_H) = \frac{\exp(\mathbf{x}_i \boldsymbol{\beta}_h)}{1 + \sum_{l=2}^{H} \exp(\mathbf{x}_i \boldsymbol{\beta}_l)}$$

$S_i \in \{1, \ldots, H\}$ ... group indicators, $i = 1, \ldots, N$

$\mathbf{x}_i$ ... row vector of regressors

$\boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_H$ ... group-specific unknown parameters

Priors, Assumptions

Assumptions:

- Set $\boldsymbol{\beta}_1 = \mathbf{0}$, which means that $h = 1$ is the baseline group and $\boldsymbol{\beta}_h$ is the effect on log-odds ratio relative to the baseline.
- Rows of $\boldsymbol{\xi}_h$ are a priori independent.
- Prior independence between $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_H$ and $\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_H$.
- Conditional on knowing $\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_H$ the observations $\mathbf{y}_1, \ldots, \mathbf{y}_N$ are mutually independent.

Priors:

- $\boldsymbol{\xi}_{h,j\,\cdot} \sim$ Dirichlet distributions with known parameters.
- $\boldsymbol{\beta}_h \sim$ normal distributions with known parameters.

MCMC Estimation

1. **Sample transition matrices $\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_H$ given $\mathbf{S}$:**

   draw each $\boldsymbol{\xi}_{h,j\cdot}$ from the Dirichlet distribution $p(\boldsymbol{\xi}_{h,j\cdot}|\mathbf{S}, \mathbf{y})$.

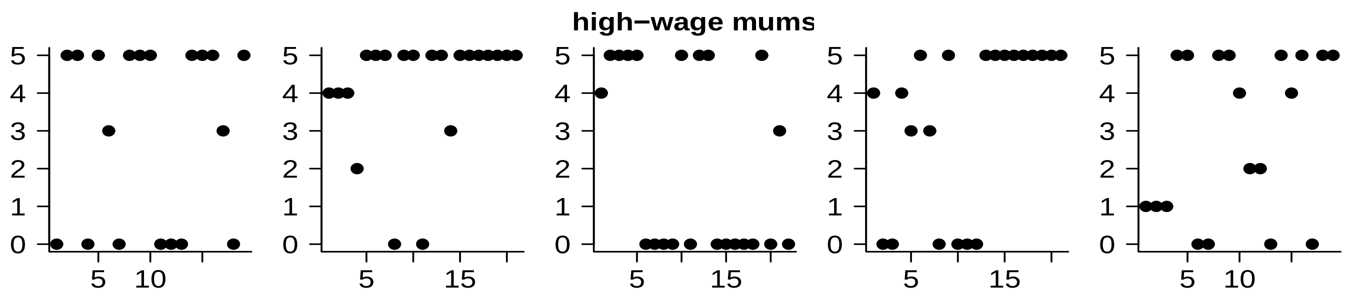
2. **Sample parameters $\boldsymbol{\beta}_2, \ldots, \boldsymbol{\beta}_H$ given $\mathbf{S}$:**

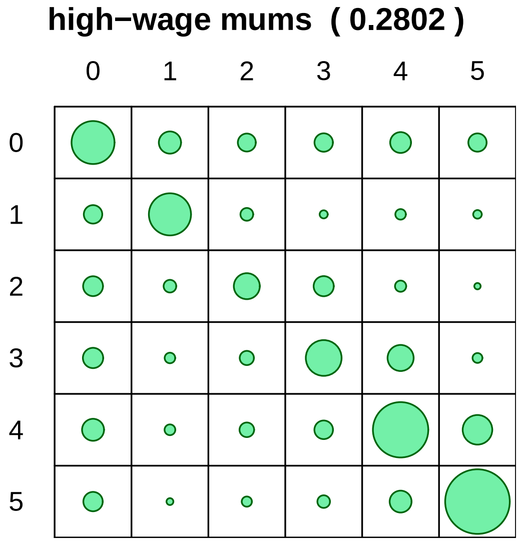
   auxiliary mixture sampling of $\boldsymbol{\beta}_h$ from the MNL involves only standard distributions (Frühwirth-Schnatter and Frühwirth 2010).

3. **Bayes' classification for each individual i:**

$$\Pr(S_i = h|\mathbf{y}_i, \mathbf{x}_i) \propto p(\mathbf{y}_i|\boldsymbol{\xi}_h) \frac{\exp\left(\mathbf{x}_i\boldsymbol{\beta}_h\right)}{1 + \sum_{l=2}^{H} \exp\left(\mathbf{x}_i\boldsymbol{\beta}_l\right)}, \quad h = 1, \ldots, H.$$

Group 1: High-Wage Mums

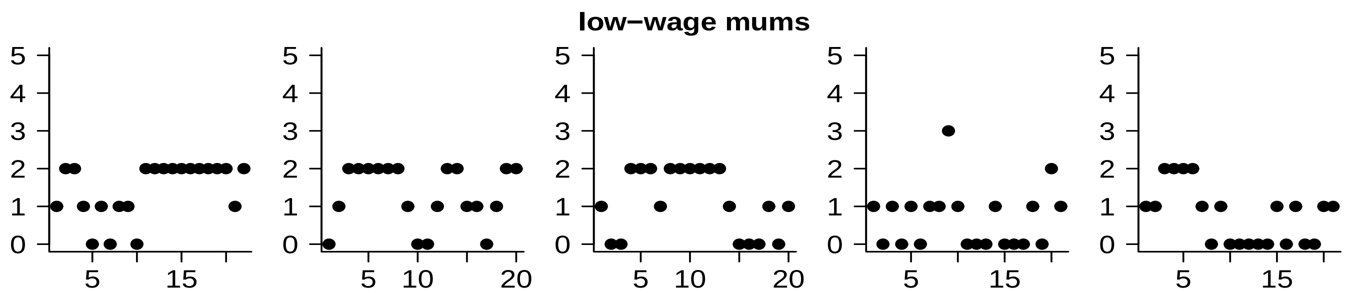


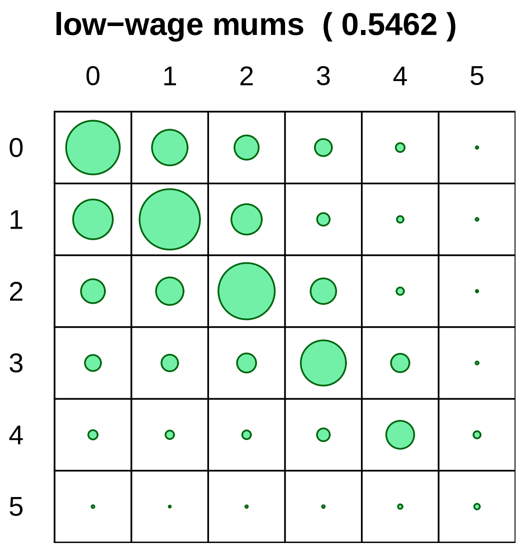


Ex post analysis:

av. age at job entry: 19.2 y.
started as white collar: 83.8 %
at least once on maternity leave: 72.6 %
number of children: $\bar{x} = 1.37$, $\tilde{x}_{0.5} = 1$

Group 2: Low-Wage Mums

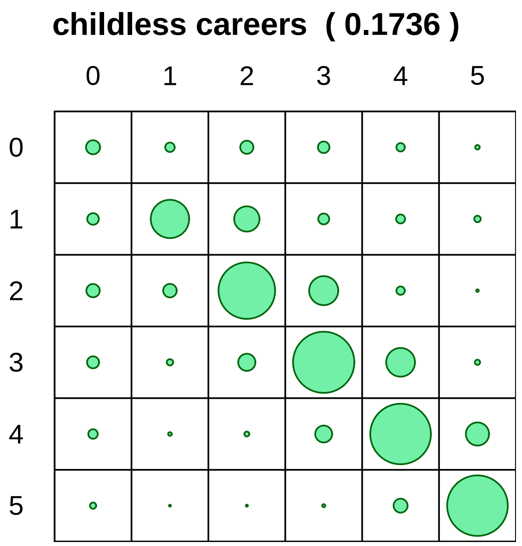




Ex post analysis:

av. age at job entry: 17.7 y.
started as white collar: 44.3 %
at least once on maternity leave: 93.4 %
number of children: $\bar{x} = 1.79$, $\tilde{x}_{0.5} = 2$

Group 3: Childless Careers



Ex post analysis:

av. age at job entry: 18.3 y.
started as white collar: 74.7 %
at least once on maternity leave: 2.6 %
number of children: $\bar{x} = 0.22$, $\tilde{x}_{0.5} = 0$

MNL for Group Membership

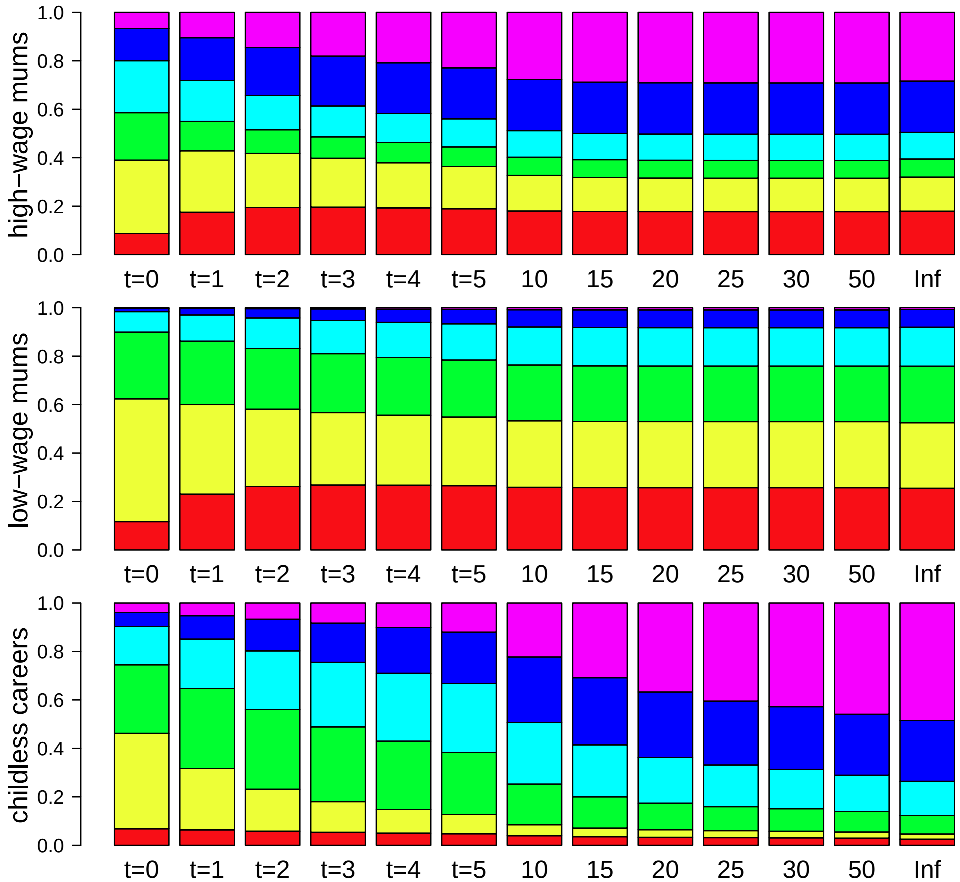
Estimates of Regression coefficients: effect on log odds (baseline: low-wage mums)

	"high-wage"	"childless careers"
Intercept	7.98821	5.05117
blue collar × maternity leave	-1.11074	-6.60941
white collar × no maternity leave	2.38276	2.66115
white collar × maternity leave	0.42969	-1.79956
Number of children	-0.22701	-0.63768
Age at start	-1.10084	-0.53595
Age at start (squared)	0.03443	0.01344

MNL for Group Membership

	"high-wage"	"childless careers"
Start in wage category 1	-0.08364	0.24558
Start in wage category 2	-0.20876	0.59434
Start in wage category 3	0.84706	1.31027
Start in wage category 4	1.80361	1.95150
Start in wage category 5	2.05103	2.58711

Long-Run Distribution



Posterior expectation of the **wage distribution** over the wage categories 0 to 5 after a period of t years in the various clusters.

Conclusions

- 3 groups of women: "high-wage mums", "low-wage-mums", "childless careers"

- variables maternity leave and number of children are very important for finding the 3 groups

- Markov chain model with logit extension allows inclusion of individual attributes

- MCMC samples from standard densities only (Gibbs)

Thank You for Your Attention!

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