Risk-parameter estimation in volatility models

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This paper introduces the concept of risk parameter in conditional volatility models and develops statistical procedures to estimate this parameter. For a given risk measure, the risk parameter is expressed as a function of the volatility coefficients and the risk of the innovation process. A two-step method is proposed to successively estimate these quantities. An alternative one-step approach, relying on a reparameterization of the model and the use of a non Gaussian QML, is proposed. Asymptotic results are established for smooth risk measures as well as for the Value-at-Risk (VaR). Asymptotic comparisons of the two approaches for VaR estimation suggest a superiority of the one-step method when the innovations are heavy-tailed. For standard GARCH models, the comparison only depends on characteristics of the innovations distribution, not on the volatility parameters. Monte-Carlo experiments and an empirical study illustrate these findings.