On moment conditions for quasi-maximum likelihood estimation of multivariate ARCH models

Paolo Zaffaroni

Joint with Marco Avarucci, Eric Beutner.

This paper questions whether it is possible to derive consistency and asymptotic normality of the Gaussian quasi-maximum likelihood estimator (QMLE) for possibly the simplest VEC-GARCH model, namely the multivariate ARCH(1) model of the BEKK form, under weak moment conditions similar to the univariate case. In contrast to the univariate specification, we show that the expectation of the loglikelihood function is unbounded, away from the true parameter value, if (and only if) the observable has unbounded second moment. Despite this non-standard feature, consistency of the Gaussian QMLE is still warranted. The same moment condition proves to be necessary and sufficient for the stationarity of the score, when evaluated at the true parameter value. This explains why high moment conditions, typically bounded sixth moment and above, have been used hitherto in the literature to establish the asymptotic normality of the QMLE in the multivariate framework.