GARCH models without positivity constraints: Exponential or Log GARCH ?

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This paper studies the probability properties and the estimation of the asymmetric log-GARCH(p, q) model. In this model, the log-volatility is written as a linear function of past values of the log-squared observations, with coefficients depending on the sign of such observations, and past log-volatility values. Conditions are obtained for the existence of solutions and finiteness of their log-moments. It is shown that the quasi-maximum likelihood estimation (QMLE) of the parameters is strongly consistent and asymptotically normal. A Lagrange-Multiplyer (LM) test is derived for testing the null assumption of a log-GARCH against more general formulations including the Exponential GARCH (EGARCH). The null assumption of an EGARCH is also tested. Portmanteau goodness-of-fit tests are developed for the log-GARCH. Simulations illustrating the theoretical results and an application to real financial data are proposed.