Limit Theory for the Largest Eigenvalues of a Sample Covariance Matrix from High-Dimensional Observations with Heavy Tails

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In this work, we study the asymptotic behavior of the eigenvalues of a sample covariance matrix XX^T, where the $n \times p$ data matrix X is obtained from observations of a high- dimensional stochastic process. Classical results in this direction often assume that the entries of X are independent and identically distributed or satisfy high moment conditions. Our goal is to weaken the moment conditions by allowing for heavy-tails, and the assumption of independent entries by allowing for dependence within the rows or columns. First we assume that the rows of X are given by independent copies of a linear process with regularly varying noise with tail index $\alpha \in (0, 2)$. It is shown that the point process based on the eigenvalues of XX^T, as the dimension p goes to infinity with n, converges in distribution to a Poisson point process with intensity measure depending on α and the sum of the squared coefficients of the underlying linear process. Therefore we obtain the joint limit distribution of the k-largest eigenvalues of the sample covariance matrix. This result is extended to random coefficient models where the coefficients of the linear processes are (possibly dependent) random variables which vary in each row of X. (This is joint work with Oliver Pfaffel and Robert Stelzer.)