Group LASSO for Structural Break Time Series

Ngai Hang Chan

Consider a structural break autoregressive (SBAR) process

$$Y_t = \sum_{j=1}^{m+1} \boldsymbol{\beta}_j^{0T} \mathbf{Y}_{t-1} I(t_{j-1} \leq t < t_j) + \varepsilon_t,$$

where $\mathbf{Y}_{t-1} = (1, Y_{t-1}, \ldots, Y_{t-p})^T$. $\boldsymbol{\beta}_j^0 = (\beta_{j0}^0, \beta_{j1}^0, \ldots, \beta_{jp}^0)^T \in \mathbf{R}^{p+1}$, $j = 1, \ldots, m+1$, $1 = t_0 < t_1 < \ldots < t_{m+1} = n+1$, $\{t_1, \ldots, t_m\}$ are change points, $\{\varepsilon_t\}$ are independent and identically distributed (i.i.d.) innovations with zero mean and unit variance. In practice, it is usually assumed that m is known and small, because a large m would involve a huge amount of computational burden in parameters estimation. By reformulating the problem in a regression variable selection context, the group least absolute shrinkage and selection operator (LASSO) is proposed to estimate an SBAR model when the number of change points m is unknown. It is shown that the number of change points and the locations of the changes can be consistently estimated from the data and the computation can be efficiently performed. Furthermore, the convergence rate of the breaks is shown to be nearly optimal. An improved practical version that incorporates group LASSO and stepwise regression variable selection technique are discussed. Simulation studies are conducted to assess the finite sample performance.

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