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Book Reviews

Norbert KUSOLITSCH (2011). **Maß- und Wahrscheinlichkeitstheorie. Eine Einführung**. Springer, Wien NewYork, 320 Seiten, ISBN 978-3-7091-0684-6. (€ 34.00)

The main object of this book is to give a thorough introduction to measure theory with applications to probability theory. In this context, this book may be rather inappropriate as a first course in probability theory; on the other hand, it provides an excellent starting point for measure-theoretic probability. As the author himself points out, the book is essentially self-contained, and tools from analysis and functional analysis are provided in the Appendix. Starting with some basic concepts of set theory in Chapter 1, the book describes the usual construction of measures on σ-algebras, based on preliminary results for measures on rings and semirings (Chapters 3 and 4). A special emphasis is put on the construction of Lebesgue-Stieltjes measures and their connection to distribution functions, which is dealt with in Chapter 6. It is worth mentioning that the book is not divided into a measure theoretic and a probabilistic part, but rather presents probabilistic applications as examples or direct consequences. In this spirit, the author presents random variables and their distributions in Chapters 7 and 8, whereas Chapter 5 gives a short summary of the basic concepts of independence and conditional probability. The construction of the integral and its basic properties, up to and including convergence theorems, is dealt with in Chapter 9. The treatment follows the classical approach, i.e. the path Beppo Levi $\rightarrow$ Fatou $\rightarrow$ dominated convergence theorem. Based on these results, the expectation is introduced via push-forward measures, and some interesting examples, like the Box-Muller algorithm are presented.

Decomposition theorems, the Radon-Nikodym theorem, product measures (including infinite products), Fubini-type theorems and the Hewitt-Savage zero-one law are presented in Chapters 10 and 11. Chapter 12 gives a brief introduction to functions of bounded variation, together with differentiation of measures. The basic discussion of measure and integration is completed in Chapter 13, where $\mathbb{L}_p$-spaces and elementary inequalities are introduced. In this chapter we encounter also uniform integrability and the dual space of $\mathbb{L}_p$. Based on the Radon-Nikodym theorem, conditional expectations and probabilities are introduced in Chapter 14. Chapter 15 deals with laws of large numbers, and also contains Birkhoff's ergodic theorem. Basic properties of discrete time martingales are introduced in Chapter 16: this includes, among others, Doob's maximal inequality and the (sub)martingal convergence theorem. The book ends with Chapter 16, where weak convergence and characteristic functions are introduced. Based on these concepts, the classical CLT and Lindeberg's CLT are presented. The book grew out of the author's lectures at the University of Vienna, and is characterized by intuitive and very detailed proofs, which is a particularly nice feature. On the whole, the book gives a very complete and self-contained introduction to measure-theoretic probability.

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