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On a Scheme of Sampling of Two Units With Inclusion Probability Proportional to Size

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Abstract: This paper introduces an unequal probability sampling without replacement scheme with inclusion probability proportional to size. This new scheme possesses some desirable properties with regard to π_i and π_{ij}, and provides a non-negative variance estimator of the Horvitz and Thompson estimator, when the values of the auxiliary variable fulfill some restrictions. On comparing the suggested scheme with some of the existing sampling schemes in respect of efficiency and stability of the variance estimator empirically, it has been observed that the performance of the scheme is satisfactory.

Zusammenfassung: Wir stellen hier einen Stichprobenplan vor, der ohne Wiederholung und mit Aufnahmewahrscheinlichkeit proportional zum Umfang arbeitet. Dieser neue Plan weist bzgl. π_i und π_{ij} einige wünschenswerte Eigenschaften auf und liefert einen nicht-negativen Varianzschätzer des Horvitz und Thompson Schätzers, falls die Hilfsvariable einige Restriktionen erfüllt. Vergleicht man empirisch den vorgeschlagenen Plan mit einigen anderen Stichprobenplänen bzgl. Effizienz und Stabilität des Varianzschätzers, so hat sich gezeigt, dass die Güte des Plans zufriedenstellend ist.

Keywords: Joint Inclusion Probability, Unequal Probability Sampling.

1 Introduction

Consider a finite population of N units, and let y_i, $i = 1, \dots, N$, denote the value for the ith unit of a certain character y. We are interested in estimating $Y = \sum_i y_i$, the total of all y-values based on a sample s of n distinct units selected from the population according to some unequal probability sampling without replacement scheme with π_i as the inclusion probability of the ith unit and π_{ij} as the joint inclusion probability of the ith and jth units. The most commonly used estimator in this connection is the Horvitz and Thompson (1952) (HT) estimator defined by

$$\hat{Y}_{HT} = \sum_{i\in s} \frac{y_i}{\pi_i} \, .$$

From Horvitz and Thompson (1952) we have $\sum_i \pi_i = n$, $\sum_{i\neq j} \pi_{ij} = (n-1)\pi_i$ and $\sum_i \sum_{j<i} \pi_{ij} = n(n-1)/2$. Sen (1953) and Yates and Grundy (1953) independently suggested an unbiased estimator of $\mathrm{var}(\hat{Y}_{HT})$ given by

$$v(\hat{Y}_{HT}) = \sum_{i\in s}\sum_{j<i} \frac{\pi_i\pi_j - \pi_{ij}}{\pi_{ij}} \left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j}\right)^2 , \qquad (1)$$

which holds for a fixed size sampling design. A sufficient condition for (1) to be always non-negative is that $\pi_{ij} < \pi_i \pi_j$, $\forall i \neq j$.

When information is available on an auxiliary character x with value x_i on unit i, considerable reduction in $\mathrm{var}(\hat{Y}_{HT})$ can be achieved by making $\pi_i = np_i$. This is of course only guaranteed if y_i/x_i is nearly constant for all i, where $p_i = x_i/X$ is the initial probability to select the ith unit and $X = \sum_i x_i$. Such a scheme is known as an inclusion probability proportional to size (IPPS or πps) sampling scheme. Brewer and Hanif (1983) and Chaudhuri and Vos (1988) have made elaborate discussions on a number of IPPS methods. But a majority of these methods are restricted to $n = 2$ only. The reasons are not far to seek. The calculation of π_{ij} becomes cumbersome for $n > 2$ and some procedures seem to be less precise than even probability proportional to size with replacement (PPS-WR) scheme. However, an IPPS sampling scheme with $n = 2$ is very useful in stratified sampling, where stratification is sufficiently *deep*, i.e. the number of strata and their sizes is such that a sample of two units per stratum meets the requirement on the total sample size. Indeed, the advantages of stratified sampling are exploited to the greatest possible extent if the number of strata is maximum and $n = 2$ is the minimum sample size to estimate the variance within each stratum (cf Chaudhuri and Vos, 1988, p. 148).

The objective of this paper is to develop an IPPS sampling scheme for $n = 2$ which possesses desirable properties and also performs well when compared to popular probability sampling schemes for a number of natural populations with different characteristics.

2 Description of the Suggested Sampling Scheme

Let us assume that $x_1, \ldots, x_N$ are known and $x_i > 0\ \forall i$. Consider a set of revised probabilities $\{P_1, \ldots, P_N\}$, where P_i is defined by

$$P_i = \frac{(1-g_i)(2p_i^2 - \lambda)}{p_i(1-2g_i)}\,, \qquad i = 1, \ldots, N\,,$$

such that $g_i = \bar{p}_h/Np_i$, $\bar{p}_h$ being the simple harmonic mean of $p_1, \ldots, p_N$, and λ is a known constant. In actual practice, we choose λ such that $\sum_i P_i = 1$. Thus, we now have

$$\lambda = \sum_i \frac{p_i}{1-2g_i} \Bigg/ \sum_i \frac{1-g_i}{p_i(1-2g_i)}\ . \tag{2}$$

It must be noted here that the computation of the revised probabilities is restricted only to situations for which $p_i^2 \geq \lambda/2$ and $g_i \leq 1/2\ \forall i$, because otherwise (1) would give negative results (see (6) below). This means that the proposed sampling scheme cannot be used when at least one of the p_i's is small compared to the others.

Our sampling scheme for $n = 2$ is defined as follows:

- Draw the first unit, say i, with revised probability P_i and without replacement, λ being given by (2).
- Draw the second unit, say j, from the remaining $(N-1)$ units with conditional probability $P_{j|i} = g_j/(1-g_i)$.

3 Inclusion Probabilities

By definition,

$$\begin{aligned}\pi_i &= P_i + \sum_{j\neq i} P_j \frac{g_i}{1-g_j} \\ &= \frac{(1-g_i)(2p_i^2-\lambda)}{p_i(1-2g_i)} + \sum_{j\neq i} \frac{g_i(2p_j^2-\lambda)}{p_j(1-2g_j)} \\ &= 2p_i - \frac{\lambda}{p_i} + g_i \sum_j \frac{2p_j^2-\lambda}{p_j(1-2g_j)}\,. \end{aligned} \tag{3}$$

From (2) we have

$$\sum_i \frac{p_i}{1-2g_i} - \frac{\lambda}{2}\sum_i \frac{1+(1-2g_i)}{p_i(1-2g_i)} = 0\,,$$

i.e.,

$$\sum_i \frac{2p_i^2-\lambda}{p_i(1-2g_i)} - \lambda \sum_i \frac{1}{p_i} = 0\,.$$

Noting that

$$g_i = \frac{1}{p_i} \bigg/ \sum_j \frac{1}{p_j}\ ,$$

we obtain

$$g_i \sum_j \frac{2p_j^2-\lambda}{p_j(1-2g_j)} - \frac{\lambda}{p_i} = 0\,. \tag{4}$$

Hence, from (3) together with (4) we get

$$\pi_i = 2p_i\,.$$

Again, by definition

$$\pi_{ij} = P_i P_{j|i} + P_j P_{i|j} \tag{5}$$

$$= \frac{g_j(2p_i^2-\lambda)}{p_i(1-2g_i)} + \frac{g_i(2p_j^2-\lambda)}{p_j(1-2g_j)}\,. \tag{6}$$

4 Properties

The important properties of the proposed sampling scheme are as follows:

(i) $\sum_i \pi_i = 2\sum_i p_i = 2$,

(ii)

$$\begin{aligned}
\sum_{j\neq i}\pi_{ij} &= \frac{(2p_i^2-\lambda)}{p_i(1-2g_i)}\sum_{j\neq i}g_j + g_i\sum_{j\neq i}\frac{2p_j^2-\lambda}{p_j(1-2g_j)} \\
&= \frac{(2p_i^2-\lambda)(1-g_i)}{p_i(1-2g_i)} - \frac{g_i(2p_i^2-\lambda)}{p_i(1-2g_i)} + g_i\sum_j \frac{2p_j^2-\lambda}{p_j(1-2g_j)} \\
&= 2p_i - \frac{\lambda}{p_i} + g_i\sum_j\frac{2p_j^2-\lambda}{p_j(1-2g_j)} \\
&= 2p_i \qquad \text{(using (4))} \\
&= \pi_i\,,
\end{aligned}$$

(iii) $\pi_i\pi_j - \pi_{ij} > 0\,, \qquad i \neq j = 1,\ldots,N.$

Following Konijn (1973, p. 253) we now have

$$\begin{aligned}
\pi_i\pi_j - \pi_{ij} &= \left(\pi_{ij} + \sum_{k\neq i\neq j}\pi_{ik}\right)\left(\pi_{ij} + \sum_{k\neq i\neq j}\pi_{jk}\right) - \pi_{ij} \\
&= \pi_{ij}\left(1 - \sum_{l\neq i\neq j}\sum_{l<k}\pi_{lk}\right) + \sum_{k\neq i\neq j}\pi_{ik}\sum_{k\neq i\neq j}\pi_{jk} - \pi_{ij} \\
&= \sum_{k\neq i\neq j}\pi_{ik}\sum_{k\neq i\neq j}\pi_{jk} - \pi_{ij}\sum_{l\neq i\neq j}\sum_{l<k}\pi_{lk}\,, \qquad (7)
\end{aligned}$$

$$\begin{aligned}
\sum_{k\neq i\neq j}\pi_{ik}\sum_{k\neq i\neq j}\pi_{jk} &= \left[\frac{2p_i^2-\lambda}{p_i(1-2g_i)}\sum_{k\neq i\neq j}g_k + g_i\sum_{k\neq i\neq j}\frac{2p_k^2-\lambda}{p_k(1-2g_k)}\right]\times \\
&\quad \left[\frac{2p_j^2-\lambda}{p_j(1-2g_j)}\sum_{k\neq i\neq j}g_k + g_j\sum_{k\neq i\neq j}\frac{2p_k^2-\lambda}{p_k(1-2g_k)}\right] \\
&= \frac{(2p_i^2-\lambda)(2p_j^2-\lambda)}{p_ip_j(1-2g_i)(1-2g_j)}\left(\sum_{k\neq i\neq j}g_k\right)^2 \\
&\quad + g_ig_j\left[\sum_{k\neq i\neq j}\frac{2p_k^2-\lambda}{p_k(1-2g_k)}\right]^2 + \pi_{ij}\sum_{k\neq i\neq j}g_k\sum_{k\neq i\neq j}\frac{2p_k^2-\lambda}{p_k(1-2g_k)}\,, \qquad (8)
\end{aligned}$$

$$\begin{aligned}
\pi_{ij}\sum_{l\neq i\neq j}\sum_{l<k}\pi_{lk} &= \pi_{ij}\sum_{l\neq i\neq j}\sum_{l<k}\left[\frac{g_k(2p_l^2-\lambda)}{p_l(1-2g_l)} + \frac{g_l(2p_k^2-\lambda)}{p_k(1-2g_k)}\right] \\
&= \pi_{ij}\sum_{k\neq i\neq j}\left[\left(\sum_{l\neq i\neq j}g_l - g_k\right)\frac{2p_k^2-\lambda}{p_k(1-2g_k)}\right]. \qquad (9)
\end{aligned}$$

Hence, from (7), (8) and (9) we have

$$\begin{aligned}\pi_i\pi_j - \pi_{ij} &= \frac{(2p_i^2-\lambda)(2p_j^2-\lambda)}{p_ip_j(1-2g_i)(1-2g_j)}\left(\sum_{k\neq i\neq j} g_k\right)^2 \\ &\quad + g_ig_j\left[\sum_{k\neq i\neq j}\frac{2p_k^2-\lambda}{p_k(1-2g_k)}\right]^2 + \pi_{ij}\sum_{k\neq i\neq j}\frac{g_k(2p_k^2-\lambda)}{p_k(1-2g_k)} \\ &> 0\,.\end{aligned}$$

Thus, an unbiased and positive estimator of the variance of the HT estimator can always be obtained under the suggested sampling scheme.

5 An Example

In order to examine how the suggested sampling scheme operates in a survey situation and possesses πps properties, we consider a small artificial population of 5 units given in Mukhopadhyay (1998, p. 220). The following table gives the values of y_i, x_i, p_i, and P_i.

y_i	123	192	212	267	290
x_i	17	18	20	22	23
p_i	0.17	0.18	0.20	0.22	0.23
P_i	0.0763	0.1240	0.2072	0.2795	0.3129

In this simple example the auxiliary variable x is not proportional to the study variable y and therefore we cannot speak of a probability proportional to size sampling design. But, it is of course a curiosity to note that in this case the proposed scheme works as all $P_i > 0$ implying that the restrictive conditions for the existence of the scheme, i.e. $p_i^2 \geq \lambda/2$ and $g_i \leq 1/2\ \forall i$ are satisfied. For $n = 2$ we compute $\pi_1 = 0.34$, $\pi_2 = 0.36$, $\pi_3 = 0.40$, $\pi_4 = 0.44$, $\pi_5 = 0.46$, $\pi_{12} = 0.0587$, $\pi_{13} = 0.0796$, $\pi_{14} = 0.0970$, $\pi_{15} = 0.1047$, $\pi_{23} = 0.0880$, $\pi_{24} = 0.1032$, $\pi_{25} = 0.1101$, $\pi_{34} = 0.1136$, $\pi_{35} = 0.1188$, and $\pi_{45} = 0.1263$, and verify that $\pi_i\pi_j - \pi_{ij} > 0\,\forall i \neq j$. Consequently, the Sen-Yates-Grundy variance estimator of the HT estimator under the suggested scheme is non-negative. The variances of $\hat{Y}_{HT}$ under the proposed scheme, the mean per unit estimator $\hat{Y} = (N/n)\sum_{i\in s} y_i$ under simple random sampling without replacement scheme and the conventional estimator $\hat{Y}_{PPS} = (1/n)\sum_{i\in s} y_i/p_i$ based on the PPSWR scheme are calculated to be 12318, 32465, and 16575, respectively. These calculations clearly show that the variance reductions of the suggested scheme compared to simple random sampling without replacement scheme and PPSWR scheme in this example are remarkable although p_i is not chosen proportional to the size of y_i. Hence, from these findings we may expect that the scheme can be safely programmed in some situations.

6 Performance of the Scheme

To compare the performance of the proposed sampling scheme with some other well known sampling schemes, we consider two different performance measures, viz.,

1. *Relative efficiency:* Here, the relative efficiency of a sampling scheme is defined as the variance ratio of the scheme and the PPSWR scheme.

2. *Stability of the variance estimator:* We accept the Hanurav (1967) criterion $\phi = \min(\pi_{ij}/\pi_i\pi_j) > \beta, \forall i \neq j$, for β sufficiently away from zero, to study stability of the variance estimator of a sampling scheme.

The following sampling plans are taken into consideration:

A Conventional estimator under PPSWR sampling scheme

B HT estimator under the sampling scheme of Brewer (1963)

C HT estimator under the sampling scheme of P. Singh (1978)

D HT estimator under the sampling scheme of Deshpande and Prabhu Ajgaonkar (1982)

E Ordered estimator of Raj (1956)

F Estimator of Rao, Hartley, and Cochran (1962)

G Unordered estimator of Murthy (1957)

H HT estimator under the suggested sampling scheme.

Three IPPS sampling procedures B, C, and D are considered for comparison with our suggested procedure H with respect to efficiency and stability of the variance estimator. To examine the efficiency of the HT estimator based on the suggested sampling scheme over other estimators based on probability proportional to size without replacement scheme, we also include the three well known estimators due to Raj, Rao-Hartely-Cochran, and Murthy in our comparison. Since a theoretical comparison is impracticable, we resort to an empirical study of the above procedures for a collection of 18 small natural populations, because a sample of size two only has been considered.

Table 1 describes source, size (N), nature of y and x, and the correlation coefficient ρ between y and x. Numerical values of the relative efficiency of the comparable sampling plans (in %), and the stability standard ϕ of the variance estimators of the plans are presented in Tables 2 and 3, respectively. Relative efficiency of a sampling scheme is computed by using the exact variance formula for the full population. On the other hand, the ϕ-value of a scheme is decided after computing the quantity $\pi_{ij}/\pi_i\pi_j$ for all $C(N, n)$ possible samples of $n = 2$ drawn from a population. Findings in these tables indicate that the suggested scheme is more efficient than other schemes for all populations, whereas its variance estimator is more stable than others for 17 populations (except population 3).

Tabelle 1: Description of populations

Pop.	Source	N	y	x	ρ
1	Cochran (1977, p. 203)	10	actual weight of peaches	estimated weight of peaches	0.97
2	D. Singh and Chaudhary (1986, p. 155)	17	no. of milch animals in survey	no. of milch animals in census	0.72
3	Konijn (1973, p. 49)	16	expenses on food	total expenses	0.95
4	Cochran (1977, p. 325)	10	no. of persons	no. of rooms	0.65
5	R. Singh and Singh Mangat (1996, p. 193, 1-14)	14	milk yield after introduction of the new feed	milk yield before introduction of the new feed	0.98
6	R. Singh and Singh Mangat (1996, p. 193, 15-27)	14	milk yield after introduction of the new feed	milk yield before introduction of the new feed	0.99
7	R. Singh and Singh Mangat (1996, p. 255)	15	tax evaded	passengers	0.59
8	Mukhopadhyay (1998, p. 114)	9	census population for 1961	census population for 1951	0.93
9	Mukhopadhyay (1998, p. 193, 1-10)	10	quantity of raw materials	no. of laborers	0.91
10	Mukhopadhyay (1998, p. 193, 11-20)	10	quantity of raw materials	no. of laborers	0.56
11	Sukhatme and Sukhatme (1970, p. 166, 1-10)	10	no. of banana bunches	no. of banana pits	0.64
12	Sukhatme and Sukhatme (1970, p. 166, 11-20)	10	no. of banana bunches	no. of banana pits	0.84
13	R. Singh and Singh Mangat (1996, p. 173, 1-13)	13	area harvested with combine	area under paddy	0.94
14	R. Singh and Singh Mangat (1996, p. 173, 14-26)	13	area harvested with combine	area under paddy	0.95
15	Yates (1953, p. 169)	17	area under wheat	total acreage of crops and grass	0.51
16	R. Singh and Singh Mangat (1996, p. 202, 1-11)	11	leaf area	leaf weight	0.95
17	R. Singh and Singh Mangat (1996, p. 202, 12-22)	11	leaf area	leaf weight	0.92
18	R. Singh and Singh Mangat (1996, p. 202, 23-33)	11	leaf area	leaf weight	0.94

Tabelle 2: Relative efficiency of different sampling plans

	Sampling Plan							
Pop.	A	B	C	D	E	F	G	H
1	100.00	112.10	112.09	111.98	111.12	112.49	112.53	114.62
2	100.00	106.73	106.72	106.72	106.27	106.24	106.69	106.77
3	100.00	107.28	107.26	107.25	106.74	107.14	107.23	108.41
4	100.00	111.64	111.66	111.56	110.90	112.50	112.24	112.66
5	100.00	108.75	108.64	108.68	107.95	108.33	108.64	113.55
6	100.00	107.92	107.46	107.87	108.11	108.33	108.84	120.33
7	100.00	107.51	107.51	107.47	107.14	107.14	107.69	107.91
8	100.00	113.85	113.82	113.71	112.53	112.49	114.34	114.76
9	100.00	112.10	112.15	112.00	111.31	112.50	112.79	113.25
10	100.00	112.94	112.56	112.77	111.50	112.00	113.01	119.05
11	100.00	113.72	113.76	113.48	111.66	112.49	113.22	114.00
12	100.00	112.30	112.38	112.15	111.25	112.50	112.71	112.99
13	100.00	107.95	107.79	107.88	108.26	108.83	109.05	113.97
14	100.00	109.15	108.88	109.05	108.59	108.33	109.42	112.30
15	100.00	106.60	106.61	106.58	106.38	106.24	106.82	106.93
16	100.00	111.14	111.03	111.02	110.05	109.59	111.18	115.32
17	100.00	112.09	111.94	111.91	110.42	110.00	111.63	117.33
18	100.00	111.86	111.69	111.69	110.34	109.99	111.53	114.91

Tabelle 3: Stability standard of different sampling schemes

	Sampling Scheme			
Pop.	B	C	D	H
1	0.54401	0.54393	0.54430	0.54740
2	0.53120	0.53118	0.53109	0.53142
3	0.52579	0.52627	0.52585	0.49623
4	0.54758	0.54754	0.54763	0.55777
5	0.53178	0.53166	0.53183	0.55757
6	0.53446	0.53372	0.53463	0.54241
7	0.54015	0.53991	0.53988	0.54510
8	0.55105	0.55105	0.55151	0.55625
9	0.52881	0.53241	0.53037	0.53510
10	0.56454	0.55505	0.56186	0.57013
11	0.54218	0.54109	0.54226	0.55003
12	0.53559	0.53793	0.53689	0.54030
13	0.51989	0.52104	0.52056	0.52779
14	0.52362	0.52511	0.52422	0.59560
15	0.53271	0.53247	0.53266	0.53538
16	0.54258	0.54286	0.54261	0.54930
17	0.55030	0.54982	0.54993	0.56442
18	0.54467	0.54179	0.54418	0.54640

7 Conclusions

On the basis of the analytical and empirical results derived in this work, we may conclude that the suggested sampling procedure is in no way inferior to some standard sampling procedures. But no general conclusion can be drawn from the empirical study as the conclusion is based on the results for 18 populations only and the gain in efficiency of the suggested scheme compared to other leading alternatives is in fact rather small. However, our empirical investigation gives an indication that the suggested sampling scheme (if it exists) compares well with other popularized schemes in terms of efficiency as well as the stability of the estimated variance.

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