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On Statistical Analysis of Compound Point Process

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Abstract: The contribution deals with a stochastic process cumulating random increments at random moments (the compound point process). First, its martingale - compensator decomposition is recalled. Then a multiplicative form of the model with the regression on covariates, simultaneously for the intensity of counting process and for the distribution of increments, is considered. Finally, a semi-parametric model is studied, the uniform consistency of estimators and the asymptotic normality of the process of residuals are proved.

Keywords: Counting Process, Compound Process, Hazard Function, Cox Model, Rate of Cumulation.

1 The Model of Compound Process

In the present paper, we consider the random process cumulating random increments at random time points, i.e. the compound (or cumulative) point process

$$C(t) = \sum_{T_i \le t} X(T_i) = \int_0^t X(s)\,\mathrm{d}N(s)\,, \qquad (C(0) = 0)\,, \tag{1}$$

where t is the time, $N(t)$ is a counting process, T_i are its random points, and $X(t)$ is a set of random variables. The model is suitable for the description of many real-world technological (e.g. shock models in reliability analysis), environmental, biological and also financial processes (especially in the field of insurance).

In the scenario considered in Volf (2000) it was assumed that each $X(t)$ was independent of the history of the process $C(s)$ up to t (on the other hand, the intensity of $N(t)$ could depend on the history). In the present paper we generalize the setting, i.e. we allow for the dependence of both process components on $\mathcal{S}(t^-)$, where $\mathcal{S}(t)$ is a corresponding filtration, i.e. a nondecreasing sequence of σ-algebras defined on the sample space of $\{N(s), Y(s), Z(s), X(s),\, 0 \le s \le t\}$; by $\mathcal{S}(t^-)$ we mean its left-continuous version, a 'history'. $Y(t)$ and $Z(t)$ are $\mathcal{S}(t^-)$ measurable predictable processes, namely $Y(t)$ is the indicator of observability of $C(t)$ and $Z(t)$ is a K-dimensional covariate process which has its values from a given set $\mathcal{Z} \subset R^K$. A review of the theory and application of counting process models is given for instance in Andersen et al. (1993). Intensity of $N(t)$ is $\lambda(t) = h(t, Z(t))Y(t)$, cumulative intensity $L(t) = \int_0^t \lambda(s)\mathrm{d}s$, $h(t,z) \ge 0$ is a hazard function.

As regards the distribution of random variables $X(t)$, we assume that the conditional distribution of $X(t)$, given $\mathcal{S}(t^-)$, can be described with a density or probability function $f(x; t, Z(t))$ and that it also possesses two first conditional moments $E(X(t)|S(t^-)) = \mu(t, Z(t))$, $\mathrm{var}(X(t)|S(t^-)) = \sigma^2(t, Z(t))$. These definitions imply that the process $N(t)$ and increments $X(t)$ depend on $\mathcal{S}(t^-)$ (and hence on each other) through $Z(t)$ and $Y(t)$.

The processes are followed throughout a time interval $[0, T]$. We assume that the indicator processes $Y(t)$ are observed fully and the covariates at least at the times when $Y(t) = 1$.

There are several ways how to specify the model of increments. For instance Scheike (1994) considered an additive regression model, $X(t, z) = \mu(t, z) + \sigma(t, z) \cdot \varepsilon$ with the focus on estimation of parametrized function μ, and, eventually, on a kernel estimation in a nonparametrized scheme. Here, we shall deal with a multiplicative form of the model, for instance $X(t, z) = X_0(t) \cdot \exp(a(z))$, which will lead to the model of Cox's type, even for the rate of growth of $C(t)$. Nevertheless, we shall formulate first some results in a quite general setting. Sections of the present paper deal with the following problems: Compensator – martingale decomposition of $C(t)$, estimate of the growth rate of $C(t)$ and its properties, multiplicative regression model both for hazard rate and increments, its semi-parametric form, method of estimation and asymptotics of estimates.

2 Compensator of Compound Process

Let us first recall the compensator – martingale decomposition of the counting process, namely $N(t) = L(t) + M(t)$, with $M(t)$ being the martingale adapted to σ-algebras $\mathcal{S}(t)$, with predictable variation process $L(t)$ (cf. for instance Andersen et al., 1993). Notice that under our assumptions on its conditional moments, $X(t)$ is conditionally orthogonal to $\mathrm{d}M(t)$, where by $\mathrm{d}M(t)$ we denote formally the increment of $M(t)$ in a small interval $[t, t + \mathrm{d}t)$. By $\mathrm{d}\langle M\rangle(t) = \mathrm{var}\{\mathrm{d}M(t)|\mathcal{S}(t^-)\}$ we mean the increment of the predictable variation process of $M(t)$, which we denote by $\langle M\rangle(t)$. While the martingales have trajectories with jumps, their predictable variation is continuous and finite provided all involved functions are bounded in $[0, T]$.

The compound process is actually a case of the marked point process (e.g. Brémaud, 1981). Let us recall here its martingale-compensator decomposition, too. The result is well-known, a proof (in a less general setting) has been included also in Volf (2000). Let us denote $X^*(t) = X(t) - \mu(t, Z(t))$, so that $\mathrm{E}(X^*(t)|\mathcal{S}(t^-)) = 0$. Then we can write

$$C(t) = \int_0^t (X^*(s) + \mu(s, Z(s)))\,\mathrm{d}N(s) = \int_0^t \mu(s, Z(s))\,\mathrm{d}L(s) + \mathcal{M}(t) \qquad (2)$$

where

$$\mathcal{M}(t) = \mathcal{M}_1(t) + \mathcal{M}_2(t) = \int_0^t X^*(s)\,\mathrm{d}N(s) + \int_0^t \mu(s, Z(s))\,\mathrm{d}M(s)\,.$$

Proposition 1. The processes $\mathcal{M}(t), \mathcal{M}_1(t), \mathcal{M}_2(t)$ are martingales adapted to σ-algebras $\mathcal{S}(t)$, the predictable variation process of $\mathcal{M}(t)$ is

$$\langle\mathcal{M}\rangle(t) = \int_0^t (\sigma^2(s, Z(s)) + \mu^2(s, Z(s)))\,\mathrm{d}L(s)\,. \qquad (3)$$

Corollary. Process $\int_0^t \mu(s, Z(s))\,\mathrm{d}L(s)$ is the compensator of process $C(t)$.

2.1 Process Characteristics

From (2) and (3) it follows that important functions characterizing the behavior of the process and of corresponding (residual) martingale are:

$h(t, z)$ – the hazard rate of counting process $N(t)$,
$k(t, z) = h(t, z) \cdot \mu(t, z)$ – the rate of cumulation of $C(t)$,
$(\mu^2(t, z) + \sigma^2(t, z)) \cdot h(t, z)$ – characterizing the variation process of residuals.

Naturally, we are also interested directly in $\mu(t, z)$, $\sigma(t, z)$, $f(y; t, z)$ describing the distribution of increments. Quite natural approach prefers to estimate the characteristics separately, i.e. from observed random points of $N_i(t)$ and from observed increments $X_i(t)$. However, we shall concentrate to the estimation of joint rate $k(t, z)$ (resp. of its integrated version), eventually in special model form cases.

Let n realizations $C_i(t) = \int_0^t X_i(s)\,\mathrm{d}N_i(s)$ of process $C(t)$, together with corresponding processes $Y_i(t), Z_i(t)$, be observed in finite interval $[0, T]$. More precisely, we observe moments of events T_{ij} of counting processes $N_i(t)$, increments $X_i(T_{ij})$, and also the paths of processes $Y_i(t), Z_i(t)$ (for $i = 1, \ldots, n$, $j = 1, \ldots, n_i = N_i(T)$). It is assumed that random variables $X_i(t)$ have the same conditional probability densities $f(y; t, z)$ and that $N_i(t)$ are characterized by the same hazard function $h(t, z)$. Now the common filtration $\mathcal{S}(t)$ contains all paths of $\{C_i(s), N_i(s), Y_i(s), Z_i(s), s \le t, i = 1, \ldots, n\}$. Counting processes $N_i(t)$ have intensity processes $\lambda_i(t) = h(t, Z_i(t))Y_i(t)$, by $L_i(t) = \int_0^t \lambda_i(s)\,\mathrm{d}s$ we denote the cumulative intensity process, $M_i(t) = N_i(t) - L_i(t)$ and $\mathcal{M}_i(t) = C_i(t) - \int_0^t \mu(s, Z_i(s))\,\mathrm{d}L_i(s)$ are martingales. If we assume the uniform boundedness of $h(t, z)$ on $[0, T] \times \mathcal{Z}$, the martingales are mutually orthogonal, i.e. $\mathrm{d}\langle M_i, M_j\rangle(t) = 0$ for $i \ne j$. The same then holds also for $\mathcal{M}_i(t)$, i.e. $\mathrm{d}\langle \mathcal{M}_i, \mathcal{M}_j\rangle(t) = 0$ for $i \ne j$, as a consequence of orthogonality of M_i, M_j and of the null probability of two increments at one moment. Finally, from this impossibility of simultaneous events it also follows that the increments of $C_i(t)$ are mutually conditionally independent, given the history of the process.

3 Multiplicative Hazard Regression Model

A general form of Cox regression model (or multiplicative, log-additive, proportional hazard model) of hazard rate of a counting process is

$$h(t, z) = h_0(t) \cdot \exp(b(z)),$$

where $b(z)$ is a response function and $h_0(t)$ is a baseline hazard rate. The most popular is the log-linear form with $b(z) = \beta \cdot z$. The estimation uses the Breslow-Crowley estimator of the cumulated baseline hazard rate $H_0(t) = \int_0^t h_0(s)\,\mathrm{d}s$

$$\hat{H}_0(t) = \int_0^t \sum_{i=1}^n \frac{\mathrm{d}N_i(s)}{\sum_{k=1}^n \exp(b(z_k(s)))Y_k(s)},$$

and the estimator of response function maximizing the logarithm of partial likelihood, $\mathcal{L}_p$, namely by the solution of equations (formally) $\mathrm{d}\mathcal{L}_p/\mathrm{d}b = 0$, i.e.

$$\frac{1}{n}\sum_i \left\{ \int_0^T b'(z_i(t)) - \frac{\sum_k b'(z_k(t)) \cdot \exp(b(z_k(t)))\, Y_k(t)}{\sum_j \exp(b(z_j(t)))\, Y_j(t)} \right\} \mathrm{d}N_i(t) = 0\,, \tag{4}$$

where (4) represents K equations (with K the dimension of covariate).

Let us further recall the conditions of stability assumed by Andersen and Gill (1982) in order to guarantee the consistency of estimation of $H_0(t)$ and β. Namely, they assume (except other conditions) the uniform (in $t \in [0, T]$) P-convergence (for $n \to \infty$)

$$\frac{1}{n}\sum_{i=1}^n \exp(b(z_i(t)))Y_i(t) \to s_0(t)\,, \tag{5}$$

where the function $s_0(t)$ is bounded and bounded away from zero.

3.1 Multiplicative Models for the Rate of Cumulation

The idea is to use a quite similar specification of the model for the main characteristics of increments. Namely, let us assume that

$$\mu(t, z) = \mu_0(t) \cdot \exp(a(z))\,.$$

Then the rate of cumulation of $C(t)$ is

$$k(t, z) = \mu_0(t) \cdot h_0(t) \cdot \exp(a(z) + b(z)) = k_0(t) \cdot \exp(c(z))\,.$$

We assume that $\mu_0(t)$, $a(z)$ are bounded functions, the same for $h_0(t)$, $b(z)$. Let us further denote $K_0(t) = \int_0^t k_0(s)\,\mathrm{d}s$ and let us assume for the function $c(z) = a(z)+b(z)$ a similar condition as (5), i.e. the uniform P-convergence:

$$\frac{1}{n}\sum_{i=1}^n \exp(c(z_i(t)))Y_i(t) \to s_0^*(t)\,, \tag{6}$$

with function s_0^* possessing the same properties as s_0 above. Here we still assume that a, b are the 'true' data-generator functions. The first consequence are the following two propositions. In them we assume that either the function $c(z)$ is known, or, at least, it is consistently estimated, uniformly in z (such a property suffices for the validity of Proposition 2).

Proposition 2. Let (6) hold and function $c(z)$ be given. Then the estimator

$$\hat{K}_0(t) = \sum_{i=1}^n \int_0^t \frac{\mathrm{d}C_i(s)}{\sum_k \exp(c(z_k(s)))Y_k(s)} \tag{7}$$

is a P-consistent, uniformly in $t \in [0, T]$, estimator of $K_0(t)$.

Proposition 3. Together with the assumptions of Proposition 2, let us assume also the existence of bounded P-limit, uniform on $[0, T]$:

$$r^*(t) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} (\sigma^2(t, z_i(t)) + \mu^2(t, z_i(t))) \exp(b(z_i(t))) Y_i(t) \,. \tag{8}$$

Then the process $\sqrt{n}(\hat{K}_0(t) - K_0(t))$ converges weakly on $[0, T]$ to a Gauss random process with independent increments and with variance function

$$w_0(t) = \int_0^t \frac{r^*(s)}{s_0^*(s)^2} \mathrm{d}H_0(s) \,.$$

The proofs are quite analogous to the proofs of Propositions 2 and 3 in Volf (2000) and are based on the form of residual process

$$\hat{K}_0 - K_0 = \sum_{i=1}^{n} \int_0^t \frac{\mathrm{d}\mathcal{M}_i(s)}{\sum_k \exp(c(z_k(s))) Y_k(s)} \,.$$

Proposition 3 suggests the way of construction of confidence bands for $K_0(t)$, provided we are able to estimate all involved functions (and then also $w_0(t)$). Simultaneously, Proposition 3 can serve as a basis for the goodness-of-fit test, because such a test compares the data with hypothetical (i.e. 'known') model functions. Such a test has been discussed for instance in Volf (2000). Naturally, the results can be modified if the functions are further specified. For instance let us imagine that $X(t, z) = X_0(t) \cdot \exp(a(z))$, so that $\mu(t, z) = \mu_0(t) \cdot \exp(a(z))$, $\sigma(t, z) = \sigma_0(t) \cdot \exp(a(z))$, and (8) simplifies, too.

Finally, let us recall that the estimate of the rate $k_0(t)$ is standardly obtained by the kernel smoothing the increments of $\hat{K}_0(t)$, and is even uniformly consistent, under certain smoothness conditions on $k_0(t)$ and a proper selection of the window-width.

In the sequel, we shall parametrize functions $a(z), b(z), c(z)$ and use the analogy with the semi-parametric Cox model analysis. We shall follow the arguments formulated for the Cox hazard regression model in Andersen et al. (1993, chapter VII.2).

3.2 Inference in Semiparametric Model

Let us consider the case that $h(t, z) = h_0(t) \cdot \exp(\beta z)$ and also $X(t, z) = X_0(t) \cdot \exp(\alpha z)$. Hence $c(z) = \gamma z$, $\gamma = \alpha + \beta$, and $\mu(t, z) = \mu_0(t) \cdot \exp(\alpha z)$, $\sigma(t, z) = \sigma_0(t) \cdot \exp(\alpha z)$.

We also assume the existence and boundedness of all model functions on $[0, T]$, i.e. of $h_0(t)$, $\mu_0(t)$, $\sigma_0(t)$, further also uniform boundedness of covariates (w.r.t. i and t, we then can skip one of additional assumptions of Andersen et al., 1993).

In the next part, by α_0, β_0, γ_0 we shall mean the 'true' values of parameters. Let us now consider the process analogous to the logarithm of Cox partial likelihood:

$$\mathcal{L}_n(\gamma) = \sum_{i=1}^{n} \int_0^T \{\log \frac{\exp(\gamma z_i(t))}{\sum_{j=1}^{n} \exp(\gamma z_j(t)) Y_j(t)}\} \mathrm{d}C_i(t) \,,$$

the K-dimensional score function, the derivative of $\mathcal{L}_n(\gamma)$, i.e.

$$\mathcal{U}_n(\gamma) = \sum_{i=1}^{n} \int_0^T \{z_i(t) - \frac{\sum_{k=1}^{n} z_k(t) \exp(\gamma z_k(t)) Y_k(t)}{\sum_{j=1}^{n} \exp(\gamma z_j(t)) Y_j(t)}\} \mathrm{d}C_i(t) \,,$$

and the derivative of $\mathcal{U}_n$, namely the $K \times K$ matrix

$$\mathcal{U}_n'(\gamma)=\sum_{i=1}^{n}\int_0^T\left\{-\frac{\sum_k z_k(t)^2\exp(\gamma z_k(t))Y_k(t)}{\sum_j\exp(\gamma z_j(t))Y_j(t)}+\left(\frac{\sum_k z_k(t)\exp(\gamma z_k(t))Y_k(t)}{\sum_j\exp(\gamma z_j(t))Y_j(t)}\right)^2\right\}\mathrm{d}C_i(t)\,.$$

Further, let us once more re-formulate already mentioned stability and regularity conditions. Let us first denote:

$$S_0(\gamma,t)=\frac{1}{n}\sum_{i=1}^{n}\exp(\gamma z_i(t))Y_i(t)\,,\quad S_1(\gamma,t)=\frac{1}{n}\sum_{i=1}^{n}z_i(t)\exp(\gamma z_i(t))Y_i(t)\,,$$

$$S_2(\gamma,t)=\frac{1}{n}\sum_{i=1}^{n}z_i(t)^2\exp(\gamma z_i(t))Y_i(t)\,,$$

$$E(\gamma,t)=\frac{S_1(\gamma,t)}{S_0(\gamma,t)}\,,\quad V(\gamma,t)=\frac{S_2(\gamma,t)}{S_0(\gamma,t)}-E(\gamma,t)^2\,.$$

Assumption A1:

1. There exists a neighborhood $\mathcal{C}$ of γ_0 and bounded functions s_0, s_1, s_2 (a scalar, vector and matrix) such that for $j=0,1,2$: $S_j(\gamma,t)\to s_j(\gamma,t)$ in probability, uniformly on $\mathcal{C}\times[0,T]$, as $n\to\infty$. Moreover, let functions s_j be continuous functions of $\gamma\in\mathcal{C}$, uniformly in $t\in[0,T]$, $s_0(\gamma,t)$ be also bounded away from zero on $\mathcal{C}\times[0,T]$, and
$$s_1=\mathrm{d}s_0/\mathrm{d}\gamma\,,\qquad s_2=\mathrm{d}^2s_0/\mathrm{d}\gamma^2\,.$$

2. Let the matrix $\Sigma=\int_0^T v(\gamma_0,t)s_0(\gamma_0,t)k_0(t)\mathrm{d}t$, where $e=s_1/s_0$ and $v=s_2/s_0-e^2$, be positive definite.

We add some more conditions of the same type, their sense will be obvious from the following context. Let us first denote, similarly as above, three functions of α, β, t

$$R_0=\frac{1}{n}\sum_{i=1}^{n}\exp((\beta+2\alpha)z_i(t))Y_i(t)\,,\qquad R_1=\frac{1}{n}\sum_{i=1}^{n}z_i(t)\exp((\beta+2\alpha)z_i(t))Y_i(t)\,,$$

$$R_2=\frac{1}{n}\sum_{i=1}^{n}z_i(t)^2\exp((\beta+2\alpha)z_i(t))Y_i(t)\,.$$

Assumption A2:

1. There exist neighborhoods $\mathcal{A}$ of α_0 and $\mathcal{B}$ of β_0 and bounded functions r_0, r_1, r_2 such that for $j=0,1,2$: $R_j(\alpha,\beta,t)\to r_j(\alpha,\beta,t)$ in probability, uniformly on $\mathcal{A}\times\mathcal{B}\times[0,T]$, as $n\to\infty$. Moreover, let functions r_j be continuous functions of $\alpha,\beta\in(\mathcal{A}\times\mathcal{B})$, uniformly in $t\in[0,T]$.

2. Let the matrix
$$\mathcal{V}=\int_0^T\left[r_2-2\frac{s_1}{s_0}r_1+s_1^2r_0\right]h_0(t)(\mu_0(t)^2+\sigma_0(t)^2)\mathrm{d}t\,,$$
evaluated at α_0,β_0, be positive definite.

Under these conditions, we are able to prove the analogs of both crucial theorems on asymptotics of estimators of Cox parameter, as formulated e.g. by Andersen et al. (1993) in Theorems VII.2.1 and VII.2.2.

Proposition 4. When $n \to \infty$, the probability that the equation $\mathcal{U}_n(\gamma) = 0$ has a solution $\hat{\gamma}$ tends to 1 and P-lim$\hat{\gamma} = \gamma_0$.

It is worth to notice that the solution is unique (if any), because functions $\mathcal{L}_n(\gamma)$ are concave strictly ($-\mathcal{U}_n'(\gamma)$ is actually a variance of certain multinomial distribution, hence positive definite, except in the case that the model is over-fitted, similarly as in the standard linear regression). The only difference in comparison with the Cox model for the hazard rate case is that now we need finite limits $r_0(t)$ and $r_1(t)$ of corresponding expressions, originated from the predictable variation process of our residual martingales $\mathcal{M}_i(t)$.

The goal of the next proposition is to show the asymptotic normality of $\sqrt{n}(\hat{\gamma} - \gamma_0)$. We have to consider two points:
1) asymptotic normality and variance of $n^{-1/2}\mathcal{U}_n(\gamma_0)$.
2) limit of $-n^{-1}\mathcal{U}_n'(\gamma)$.
As regards 1), first notice that

$$\mathcal{U}_n(\gamma_0) = \sum_{i=1}^{n} \int_0^T \left\{ z_i(t) - \frac{\sum_{k=1}^n z_k(t) \exp(\gamma_0 z_k(t)) Y_k(t)}{\sum_{j=1}^n \exp(\gamma_0 z_j(t)) Y_j(t)} \right\} \mathrm{d}\mathcal{M}_i(t)$$

and also that $\mathrm{d}\langle \mathcal{M}_i \rangle(t) = \exp((\beta + 2\alpha) z_i(t)) Y_i(t) h_0(t) (\mu_0(t)^2 + \sigma_0(t)^2)\mathrm{d}t$. Hence, the asymptotic variance of $n^{-1/2}\mathcal{U}_n(\gamma_0)$ equals the limit of

$$\mathcal{V}_n = \frac{1}{n} \sum_{i=1}^{n} \int_0^T \left[R_2 - 2\frac{S_1}{S_0} R_1 + S_1^2 R_0 \right] h_0(t) (\mu_0(t)^2 + \sigma_0(t)^2) \mathrm{d}t\,,$$

evaluated at estimated parameters. This limit equals $\mathcal{V}$, due assumptions A1, A2, and the consistency of both $\hat{\beta}$ (Cox model parameter) and $\hat{\gamma}$, with $\hat{\alpha} = \hat{\gamma}/\hat{\beta}$ (provided $\hat{\beta} \neq 0$). It is seen that in order to get fully empirical version $\hat{\mathcal{V}}_n$ as a consistent estimator of $\mathcal{V}$, we need also an uniformly in $[0, t]$ consistent estimator of σ_0, while h_0, k_0 and $\mu_0 = k_0/h_0$ are already available (the latter at t where estimated $h_0(t)$ is positive).

The second point leads, again by the compensator - martingale decomposition, to the expression

$$-n^{-1}\mathcal{U}_n'(\gamma) \sim \Sigma_n(\gamma) = \int_0^T V(\gamma, t) S_0(\gamma, t) \mathrm{d}K_0(t)$$

(martingale part vanishes asymptotically and uniformly, again due the uniform boundedness of the predictable variation process of residual martingales). And when we consider γ between γ_0 and the estimate $\hat{\gamma}$ (in Taylor expansion of $\mathcal{U}_n(\gamma)$ at γ_0), then $\Sigma_n(\gamma)$, thanks assumptions A1 and A2, tends to Σ. The same then holds also for the fully empirical version

$$\hat{\Sigma}_n = \int_0^T V(\hat{\gamma}, t) S_0(\hat{\gamma}, t) \mathrm{d}\hat{K}_0(t)\,.$$

Therefore, we may state the following proposition (the analog to Theorem VII.2.2 of Andersen et al. (1993)).

Proposition 5. When $n \to \infty$, $n^{1/2}(\hat{\gamma} - \gamma_0) \to \mathcal{N}(0, \Sigma^{-1}\mathcal{V}\Sigma^{-1})$ in distribution. The asymptotic variance matrices can be estimated consistently with the aid of $\hat{\Sigma}_n$, $\hat{\mathcal{V}}_n$.

4 Conclusion

The main purpose of the paper was to propose a multiplicative form of regression model for the cumulative process and to study the methods of its evaluation. The obtained results can be summarized to the following points:

In general, the estimator of function $c(z)$ is available from the normal equations similar to (4). Estimates of $a(z)$ and $\mu_0(t)$ follow immediately. If response functions are parametrized, consistent and asymptotically normal estimates of parameters exist, under proper stability conditions. However, the estimates of variance $\sigma(t, z)$ (or $\sigma_0(z)$, respectively) are not available and should be obtained in a standard way, from the analysis of increments. The case simplifies if the increments are modelled via a distribution with one parameter, for instance in the case of exponential distribution, when $\sigma = \mu = \mu_0(t) \cdot \exp(\alpha z)$, or in the case of integer-valued increments described by a Poisson model (then $\sigma^2 = \mu$). Finally, it has also been shown that there exists a potentially consistent and asymptotically normal estimator of the cumulated baseline rate $K_0(t)$ of process $C(t)$.

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