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Instrumental Weighted Variables

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Abstract: A motivation for the classical Instrumental Variables and the reasons for here-proposed way of their robustification are discussed. The conditions for the $\sqrt{n}$-consistency, the existence of Bahadur representation and the asymptotic normality of the robustified estimator are given.

Keywords: Robustness, Instrumental Variables, Weighting.

1 Introduction, Instrumental Variables, and Objectives

Let $\mathcal{N}$ denote the set of all positive integers, $\mathcal{R}$ the real line and $\mathcal{R}^p$ the p-dimensional Euclidean space. The linear regression model given as

$$Y_i = X_i'\beta^0 + e_i = \sum_{j=1}^{p} X_{ij}\beta_j^0 + e_i\,, \qquad i = 1, \ldots, n$$

will be considered. We shall assume that:
C1 The sequence $\{(X_i', e_i)'\}_{i=1}^{\infty} \subset \mathcal{R}^{p+1}$ is sequence of independent and identically distributed (iid) random variables with absolutely continuous distribution function (d.f.) $F_{X,e}(x, v)$. Moreover, $\mathrm{E}\{(X', e)' \cdot (X', e)\}$ is a positive definite matrix and the conditional density $f_{e|X}(v|X_1 = x)$ is uniformly in x bounded.

If orthogonality condition is broken, i.e. if $\mathrm{E}\{e_i|X_i\} \neq 0$ (an example follows), the Ordinary Least Squares are inconsistent, as the following relations show:

$$\hat{\beta}^{(OLS,n)} = \beta^0 + \left(\frac{1}{n}\sum_{k=1}^{n} X_k X_k'\right)^{-1} \frac{1}{n}\sum_{i=1}^{n} X_i e_i \quad \text{and} \quad \operatorname*{plim}_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} X_i e_i = \mathrm{E}X_1 e_1\,. \quad (1)$$

Let us assume the model with lagged explanatory variables (Judge et al., 1985; or Víšek, 1998) in the simplest version, i.e. with the geometric structure of coefficients,

$$Y_t = \gamma \sum_{j=1}^{\infty} \lambda^{j-1} x_{t-j+1} + e_t\,, \qquad t = \ldots, -1, 0, 1, \ldots, T\,,$$
$$\mathrm{E}e_t = 0\,, \quad \mathrm{E}e_t^2 = \sigma^2 \in (0, \infty)\,, \quad \text{and} \quad \lambda \in (-1, 1)\,. \quad (2)$$

Clearly, we are not able to estimate coefficients γ and λ directly, hence writing model for $t-1$, multiplying it by λ and subtracting from (2), we obtain

$$Y_t = \lambda Y_{t-1} + \gamma x_t + e_t - \lambda e_{t-1} = \lambda Y_{t-1} + \gamma x_t + u_t\,. \quad (3)$$

Now, the "explanatory" variable Y_{t-1} is correlated with the error term u_t and then (1) indicates that *OLS* estimate of regression coefficients of model (3) is inconsistent. Another frequently given example of failure of the orthogonality condition is the case of measuring explanatory variables with random error (see Carroll et al., 1995; or Judge et al., 1985, Víšek (1998)). To cope with such situations the method of Instrumental Variables (IV) was proposed (e.g. Carroll et al., 1995, Judge et al., 1985) as the solution of normal equations

$$\sum_{i=1}^{n} Z_i \left(Y_i - X_i'\beta\right) = 0\,, \tag{4}$$

where the elements of the sequence $\{Z_i\}_{i=1}^{\infty}$ are usually called instruments. At the end of the last century IV became more or less a standard tool in many case studies of panel data, especially in econometrics. Of course, this is only one of possibilities how to cope with it. Another is e.g. the Total Least Squares, see Van Huffel (2004). Simplifying a bit, we may say that decision which method to use in such a situation depends on the fact whether for given data the error term of the regression model represents the measurement error of the response variable, as in technical or natural sciences usually does, or if some explanatory variables which are not available or not to easy to measure etc. are included into the error term, as it is usual in social sciences to assume. Naturally, to obtain then unbiased estimates of regression coefficients we have to believe that this implicit segment of explanatory part of model (creating together with the possible measurement error of response variable the error term) is orthogonal to the segment which is explicitly given in the model, see Chatterjee and Hadi (1988).

In our example (with lagged variables) we can use as the instrument for Y_{t-1} the value x_{t-1} (which is independent from $u_t = e_t - \lambda e_{t-1}$) or a linear combination $\sum_{j=1}^{k} \alpha_j x_{t-j}$ for some k (which is also independent from u_t) because we assume that Y_t depends of the x_{t-j}'s for $j = 0, 1, \ldots$. For the situation with measurement error consult please Carroll et al. (1995). Moreover, a lot of recommendations how to select the instruments for explanatory variables were established, see e.g. Arellano and Bond (1991), Arellano and Bover (1995), Bowden and Turkington (1984), or Sargan (1988) (and for examples of implementation see for SAS - Der and Everitt (2002), for R and S-PLUS - Fox (2002)).

Since the system of equations (4) is an analogy of normal equations for the OLS, the estimate by means of Instrumental Variables suffer by the lack of robustness both in the case of presence of outlier(s) among $Y_i - X_i'\beta$, $i = 1, \ldots, n$, as well as in the case of presence of leverage point(s) among Z_i's. That is why the paper offers a proposal of robustified version of IV based on the idea of implicit weighting the residuals, as it was used by the Least Weighted Squares (LWS), see Víšek (2001).

2 The Least Weighted Squares

Hettmansperger and Sheather (1992), when processing the Engine Knock Data (Mason et al., 1989), were surprised that the result of robust estimation may be considerably influenced by a small change of data. Although their result was due to a bad algorithm they employed for the Least Median of Squares (Rousseeuw, 1984), it began the studies looking for an explanation of this fact. For the correction see Víšek (1994) where the faster algorithm by Boček and Lachout (1995) gave (much) smaller value of the minimized functional. Finally, it was given in Víšek (1996a) and Víšek (2000b). The explanation demonstrated that even an arbitrarily small change of data may really cause - in the case of the estimators with high breakdown point - a change of the estimates of regression coefficients as large as you want. A numerical example was offered already in Víšek (1994); the result of processing Engine Knock Data by the Least Trimmed Squares (by the algorithm, performing a complete search and hence giving the precise value of the estimator) demonstrated that the small change in data caused very large change of the estimate. The asymptotic representation of the difference of the estimates for the all available data and for a subsample obtained by trimming-off even only one observation in the case of the M-estimators and the LTS indicate that this difference for the estimators with the discontinuous weight functions may be rather large (although bounded in probability), see Víšek (1992, 1996b, 2000a, 2002a), compare also Chatterjee and Hadi (1988)).

When processing the panel data, we cannot (generally) trim-off any observation, since it could destroy (or at least considerably damage) the correlation structure of disturbances and/or of explanatory variables. Moreover, trimming-off some observation may mask the heteroscedasticity of data (compare the processing of the same data sets in Rousseeuw and Leroy (1987) and in Chatterjee and Hadi (1988)). It indicates that the weighting down the residuals may be sometimes reasonable solution. Of course, if all observations obtain nonzero weights, the breakdown point becomes zero. However, it is due to theoretical possibility of shifting some observations into the infinity. In fact, if the outliers as well as leverage points obtain ("sufficiently") small weights, we are able to cope with them; of course, it requires to experiment with weight function w. However, simple examples demonstrate that the weighting which is based on an external rule (e.g. a geometric rule) may end in a considerable loss of information or can establish a misleading identification of underlying model. Then a straightforward idea may be to employ an implicit weighting (we shall indicate the fact that the weighting is implicit by the order of words in the name of method).

For any $\beta \in \mathcal{R}^p$, $r_i(\beta) = Y_i - X_i'\beta$ denotes the i-th residual and $r^2_{(h)}(\beta)$ the h-th order statistic among the squared residuals, i.e. we have

$$r^2_{(1)}(\beta) \leq \cdots \leq r^2_{(n)}(\beta)\,.$$

Definition 2.1 *For any $n \in \mathcal{N}$ let $w_1 \geq \cdots \geq w_n$, $w_i \in [0,1]$, be some weights. Then*

$$\hat{\beta}^{(LWS,n,w)} = \mathop{\mathit{arg\,min}}_{\beta \in R^p} \sum_{i=1}^{n} w_i r^2_{(i)}(\beta) \tag{5}$$

will be called the Least Weighted Squares (see Víšek, 2001; and also Víšek, 2002b).

The weights are usually generated by a weight function w with following properties (compare Hájek and Šidák, 1967):
C2 Weight function $w : [0,1] \to [0,1]$ is absolutely continuous and nonincreasing, with the derivative $w'(\alpha)$ bounded from below by $-L$, $w(0) = 1$.

Putting then $w_i = w(i - 1/n)$, we can rewrite (5) into the form (see also Čížek, 2002, where the estimator is called the Smoothed Least Trimmed Squares)

$$\hat{\beta}^{(LWS,n,w)} = \mathop{\arg\min}_{\beta \in R^p} \sum_{i=1}^{n} w\left(n^{-1}(i-1)\right) r^2_{(i)}(\beta)\,.$$

See also Koul (1992); Koul and Ossiander (1994) and references given there, for (much more) general estimators of similar type, i.e. weighted by a function of ranks, considered in the autoregression framework. Now, following Hájek and Šidák (1967), let us define ranks. For any $i = 1, \ldots, n$ put $\pi(\beta, i) = j \in \{1, \ldots, n\}$ iff $r_i^2(\beta) = r_{(j)}^2(\beta)$. Then we arrive at

$$\hat{\beta}^{(LWS,n,w)} = \arg\min_{\beta \in R^p} \sum_{i=1}^{n} w\left(n^{-1}(\pi(\beta, i) - 1)\right) r_i^2(\beta)\,.$$

It is then easy to show that $\hat{\beta}^{(LWS,n,w)}$ is (one of) solution(s) of the normal equations

$$\sum_{i=1}^{n} w\left(n^{-1}(\pi(\beta, i) - 1)\right) X_i\,(Y_i - X_i'\beta) = 0\,. \tag{6}$$

Denoting by $I\{A\}$ the indicator of the set A, for any $\beta \in \mathcal{R}^p$ and any $r \in \mathcal{R}$ define the empirical distribution function (e.d.f.) of the absolute values of residuals as

$$F_\beta^{(n)}(r) = \frac{1}{n}\sum_{j=1}^{n} I\left\{\omega \in \Omega : |r_j(\beta)| < r\right\} = \frac{1}{n}\sum_{j=1}^{n} I\left\{\omega \in \Omega : |e_j - X_j'\beta| < r\right\}\,. \tag{7}$$

Further, denoting $|r_i(\beta)| = a_i(\beta)$, one can easily verify that the order statistics of the absolute values of residuals $a_{(i)}(\beta)$'s and the order statistics of the squared residuals $r_{(i)}^2(\beta)$'s assign to given fix observation the same rank, i.e. the residual of given fix observation is in the sequences $r_{(1)}^2(\beta) \le \cdots \le r_{(n)}^2(\beta)$ and $a_{(1)}(\beta) \le \cdots \le a_{(n)}(\beta)$ on the same position. It is then straightforward that due to the left-continuity of e.d.f. (7), we have

$$F_\beta^{(n)}(a_{(\pi(\beta,i))}(\beta)) = F_\beta^{(n)}(|r_i(\beta)|) = n^{-1}(\pi(\beta, i) - 1)$$

and so the normal equations (6) can be written as

$$\sum_{i=1}^{n} w\left(F_\beta^{(n)}(|r_i(\beta)|)\right) X_i\,(Y_i - X_i'\beta) = 0\,. \tag{8}$$

3 Instrumental Weighted Variables

Robustifying (4) in the analogy with (8) we define

Definition 3.1 *For any sequence of random vectors* $\{Z_i\}_{i=1}^{\infty} \subset \mathcal{R}^p$ *the solution(s) of the normal equation*

$$\sum_{i=1}^{n} w\left(F_\beta^{(n)}(|r_i(\beta)|)\right) Z_i(Y_i - X_i'\beta) = 0 \tag{9}$$

is called the Instrumental Weighted Variables (IWV) estimator and denoted by $\hat{\beta}^{(IWV,n,w)}$.

For the consistency of the IWV we will need some further assumptions.
C3 The instrumental variables $\{Z_i\}_{i=1}^{\infty} \subset \mathcal{R}^p$ are iid with d.f. $F_Z(z)$. Moreover, they are independent from the sequence $\{e_i\}_{i=1}^{\infty}$. Further, the joint d.f $F_{X,Z}(x, z)$ is absolutely continuous. Finally, $\mathrm{E}\{w(F_{\beta^0}(|e_1|))Z_1X_1'\}$ as well as $\mathrm{E}Z_1Z_1'$ are positive definite (compare C3 with Víšek, 1998, 1998b, where we considered instrumental M-estimators and the discussion of assumptions for M-instrumental variables was given) and there is $q > 1$ so that $\mathrm{E}\{\|Z_1\| \cdot \|X_1\|\}^q < \infty$.

The conditions under which the IWV are consistent, requires some other notations. For any $\beta \in \mathcal{R}^p$ the d.f. of the product $\beta' Z_1 X_1' \beta$ will be denoted $F_{\beta' Z_1 X_1' \beta}(u)$, i.e.

$$F_{\beta' Z_1 X_1' \beta}(u) = P(\beta' Z_1 X_1' \beta < u)$$

and as in previous, the corresponding e.d.f. will be denoted $F^{(n)}_{\beta' Z X' \beta}(u)$, so that

$$F^{(n)}_{\beta' Z X' \beta}(u) = \frac{1}{n} \sum_{j=1}^{n} I\{\beta' Z_j X_j' \beta < u\} = \frac{1}{n} \sum_{j=1}^{n} I\{\omega \in \Omega : \beta' Z_j(\omega) X_j'(\omega) \beta < ut\}\,.$$

For any $\lambda \in \mathcal{R}^+$ and any $a \in \mathcal{R}$ put

$$\gamma_{\lambda,a} = \sup_{\|\beta\|=\lambda} F_{\beta' Z X' \beta}(a)\,. \tag{10}$$

Notice, that due to the fact that the surface of the ball $\{\beta \in \mathcal{R}^p, \|\beta\| = \lambda\}$ is compact, there is $\beta_\gamma \in \{\beta \in \mathcal{R}^p, \|\beta\| = \lambda\}$ such that

$$\gamma_{\lambda,a} = F_{\beta_\gamma' Z X' \beta_\gamma}(a)\,.$$

For any $\lambda \in \mathcal{R}^+$ let us denote

$$\tau_\lambda = - \inf_{\|\beta\| \le \lambda} \beta' \mathrm{E}\left[Z_1 X_1' \cdot I\{\beta' Z_1 X_1' \beta < 0\}\right] \beta\,. \tag{11}$$

C4 There is $a > 0$, $b \in (0, 1)$ and $\lambda > 0$ so that (for $\gamma_{\lambda,a}$ and τ_λ see (10) and (11))

$$a \cdot (b - \gamma_{\lambda,a}) \cdot w(b) > \tau_\lambda\,.$$

C5 There is the only solution of

$$\beta' \mathrm{E}\left[w\left(F_\beta(|r_1(\beta)|)\right) Z_1 \left(e_1 - X_1' \beta\right)\right] = 0 \tag{12}$$

namely β^0 (the equation (12) is assumed to be vector equation in $\beta \in \mathcal{R}^p$, of course).

Lemma 3.1 *Let the conditions* **C1**, ..., **C5** *be fulfilled. Then any sequence* $\{\hat{\beta}^{(IWV,n,w)}\}_{n=1}^{\infty}$ *of the solutions of normal equations (9) is weakly consistent.*

Due to the limited space of paper we are not able to give proofs of the results (they need about 65 pages, see Víšek, 2005a, where all proof are given in details), hence we offer only a sketch of proof of $\sqrt{n}$-consistency of $\hat{\beta}^{(IWV,n,w)}$ (given below, which is shortest) to hint the character of ideas which are the proofs based on. They are mainly a long chain of small technicalities with one exception - Skorohod's embedding into Wiener process. $F_e(r)$ and $f_e(r)$ stay for for the marginal of $F_{X,e}(x, v)$ and the marginal density, respectively.

Naturally, for the $\sqrt{n}$-consistency of $\hat{\beta}^{(IWV,n,w)}$ we need to enlarge a bit the conditions.

NC1 The density $f_{e|X}(r|X_1 = x)$ is uniformly with respect to x Lipschitz of the first order. Moreover, $f_e'(r)$ exists and is bounded in absolute value.

NC2 The derivative $w'(\alpha)$ of the weight function is Lipschitz of the first order.

Lemma 3.2 *Let the conditions* **C1***, ...,* **C5***,* **NC1** *and* **NC2** *be fulfilled. Then any sequence* $\{\hat{\beta}^{(IWV,n,w)}\}_{n=1}^{\infty}$ *of the solutions of normal equations (9) is* $\sqrt{n}$*-consistent.*

Sketch of proof: We will need the following
Assertion 3.1 Let condition **C1** hold and fix arbitrary $\varepsilon > 0$ and put $K = \sqrt{8/\varepsilon} + 1$. Then there is $n_\varepsilon \in \mathcal{N}$ so that for all $n > n_\varepsilon$

$$P\left(\left\{\omega \in \Omega : \sup_{v \in \mathcal{R}^+} \sup_{\beta \in \mathcal{R}^p} \sqrt{n} \left| F_\beta^{(n)}(v) - F_\beta(v) \right| < K \right\}\right) > 1 - \varepsilon \, .$$

For the proof of assertions see Víšek (2005b). Rewriting (9) as

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} w\left(F_\beta^{(n)}(|r_i(\beta)|)\right) Z_i e_i = \frac{1}{n} \sum_{i=1}^{n} w\left(F_\beta^{(n)}(|r_i(\beta)|)\right) Z_i X_i^T \cdot \sqrt{n}\left(\beta - \beta^0\right) ,$$

we can employ Assertion 3.1 and arrive at

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} w\left(F_\beta(|r_i(\beta)|)\right) Z_i e_i + R_n^{(1)} = \frac{1}{n} \sum_{i=1}^{n} \left[w\left(F_\beta(|r_i(\beta)|)\right) Z_i X_i^T + R_n^{(2)} \right] \cdot \sqrt{n}(\beta - \beta^0) , \tag{13}$$

where

$$R_n^{(1)} = R_n^{(1)}(\beta, X, Z, e) \quad \text{with} \quad \sup_{\beta \in \mathcal{R}^p} \left\| \mathcal{R}_n^{(1)}(\beta, X, Z, e) \right\| = \mathcal{O}_p(1) \tag{14}$$

and

$$R_n^{(2)} = R_n^{(2)}(\beta, X, Z, e) \quad \text{with} \quad \sup_{\beta \in \mathcal{R}^p} \left\| R_n^{(2)}(\beta, X, Z, e) \right\| = o_p(1) \, . \tag{15}$$

Utilizing standard steps of functional analysis we can modify (13) into

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left\{ w'(F_{\beta^0}(|r_i(\beta)|)) \cdot [F_\beta(|r_i(\beta)|) - F_{\beta^0}(|r_i(\beta)|)] + R_{ni}^{(3)} \right\} Z_i e_i$$

$$+ \frac{1}{\sqrt{n}} \sum_{i=1}^{n} w\left(F_{\beta^0}(|r_i(\beta)|)\right) Z_i e_i + R_n^{(1)}$$

$$= \frac{1}{n} \sum_{i=1}^{n} \{ w'(F_{\beta^0}(|r_i(\beta)|)) \cdot [F_\beta(|r_i(\beta)|) - F_{\beta^0}(|r_i(\beta)|)] + R_{ni}^{(4)} \} \cdot Z_i X_i^T \cdot \sqrt{n}\left(\beta - \beta^0\right)$$

$$+ \left[\frac{1}{n} \sum_{i=1}^{n} w\left(F_{\beta^0}(|r_i(\beta)|)\right) Z_i X_i^T + R_n^{(2)} \right] \cdot \sqrt{n}\left(\beta - \beta^0\right) \tag{16}$$

where, of course, for all rests $R_n^{(j)}$ relations similar to (14) or (15) hold. Then a long chain of steps proving that we may substitute in the previous expression the residual $r_i(\beta)$ by $r_i(\beta^0) = e_i$ follows. The full version of proof requires 11 pages which however contain mostly small standard steps of approximating one expression by other, equivalent in probability. It allows to show that the left-hand-side of (16) is $\mathcal{O}_p(1)$ and that the terms of the right-hand-side converge in probability to regular matrix times $\sqrt{n}\,(\beta - \beta^0)$. The proof is then concluded by the employment of
Assertion 3.2 Let for some $p \in \mathcal{N}$, $\{\mathcal{V}^{(n)}\}_{n=1}^{\infty}$, $\mathcal{V}^{(n)} = \{v_{ij}^{(n)}\}_{i=1,\ldots,p}^{j=1,\ldots,p}$ be a sequence of

$(p \times p)$ matrices such that for $i = 1, \ldots, p$ and $j = 1, \ldots, p$ $\lim_{n\to\infty} v_{ij}^{(n)} = q_{ij}$ in probability, where $Q = \{q_{ij}\}_{i=1,\ldots,p}^{j=1,\ldots,p}$ is a fixed non-random regular matrix. Moreover, let $\{\theta^{(n)}\}_{n=1}^{\infty}$ be a sequence of p-dimensional random vectors such that

$$\exists\,(\varepsilon > 0)\,\forall\,(K > 0)\ \limsup_{n\to\infty} P\left(\|\theta^{(n)}\| > K\right) > \varepsilon\,.$$

Then

$$\exists\,(\delta > 0)\,\forall\,(H > 0) \quad \Longrightarrow \quad \limsup_{n\to\infty} P\left(\left\|\mathcal{V}^{(n)}\theta^{(n)}\right\| > H\right) > \delta$$

(for the proof of assertions see Víšek, 2002a).

Of course, for the asymptotic representation we need again to strengthen our conditions on the underlying d.f. Let $g(z)$ be the density of the d.f. $G(z) = P(e_1^2 < z)$.
AC1 For any $a \in \mathcal{R}$ there is $\Delta(a) > 0$ so that $\inf_{z\in(0,a+\Delta(a))} g(z) > L_{g,a} > 0$.
AC2 There is $q > 1$ so that $\mathrm{E}\,|e_1|^{2q} < \infty$.

Lemma 3.3 *Let the conditions* **C1***, ...,* **C5***,* **NC1***,* **NC2***,* **AC1***, and* **AC2** *hold and let* $Q = E\{w(F_{\beta^0}(|e|))Z_1X_1'\}$*. Then*

$$\sqrt{n}\left(\hat{\beta}^{(LWS,n,w)} - \beta^0\right) = Q^{-1} \cdot \frac{1}{\sqrt{n}} \sum_{i=1}^{n} w\left(F_{\beta^0}(|e_i|)\right) \cdot Z_i e_i + o_p(1)\,.$$

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