A Simple Encompassing Test for the Deterministic and Bilinear Unit Root Models

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Abstract: A new parameters' encompassing test is proposed for deciding between the deterministic unit root processes with a structural break and the bilinear unit root model without such break. The test consists in testing three sets of hypotheses regarding parameters in a simple regression model. The test uses the t-ratio and F-statistics, of non-trivial distributions under the null hypothesis. The finite sample distributions for the relevant statistics are tabulated and the asymptotic distribution of the F-test is derived. The test has been applied for the daily stock price indices for 66 countries, for the period 1992-2001. The results support the conjecture that the bilinear model dominates the structural break model more often than the other way around. Also, it is likely that in practical applications the bilinear unit root process might be mistaken for the deterministic unit root process with a structural break.

1 The Model

The paper considers two particular stochastic processes embedded within the following general process:

$$u_t = d_t + y_t, \quad y_t = \rho_t y_{t-1} + v_t \tag{1}$$

where d_t is a deterministic part, $E\left(y_0^p\right)^2 < \infty$, $v_t = \delta(L)\varepsilon_t = \sum_{j=0}^{\infty} \delta_j \varepsilon_{t-j}$ and $\sum_{j=0}^{\infty} j|\delta_j| < \infty$ (see Ng and Perron, 2001), ρ_t is series of (possibly degenerated) random variables, $\varepsilon_t \sim IID(0, \sigma^2)$ and $t = 1, 2, \ldots, T$. The deterministic part in (1) is given as:

$$d_t = const. + \gamma B_t, \tag{2}$$

where B_t is a variable indicating a nonlinear change in the deterministic part of the process. The most commonly used form of the break is either the 'slope':

$$B_t = \begin{cases} (t - T_B), & \text{for } t > T_B, \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

or the 'step':

$$B_t = \begin{cases} 0, & \text{for } t < T_B, \\ 1, & \text{otherwise,} \end{cases} \tag{4}$$

and T_B indicates the position of the beginning of the structural break in the series. A number of other, more complex, specifications of B_t is also possible (see Vogelsang and Perron, 1998; Harvey and Mills, 2002).

As far as the unit root part in (1) is concerned, the following terminology is introduced:

(a) The unit root process has a *deterministic* unit root and is *linear* if in (1) $\rho_t = 1$ and in (2) $\gamma = 0$.

(b) The unit root process is *deterministic* unit root and is *nonlinear* if $\rho_t = 1$ and $\gamma \neq 0$.

(c) The unit root process has a *stochastic* unit root if ρ_t is a non-degenerated random variable with the expected value of one. In this paper we only consider the stochastic unit root process without a deterministic part, that is where $\gamma = 0$.

The first two processes, (a) and (b), constitute a family of the deterministic unit root (*URD*) processes. The literature on testing and evaluation of the *URD* processes is well developed. In particular, the nonlinear *URD* models (and such models are considered in this paper) are analysed in Kim et al. (2000); Ng and Perron (2001); Perron (1989, 1990, 1997); Vogelsang and Perron (1998); Zivot and Andrews (1992). Among the stochastic unit root processes, this paper focuses on the bilinear unit root (*URB*) process of the first order (see Charemza et al., 2005), defined as:

$$\rho_t = 1 + b\varepsilon_{t-1}, \ b \neq 0, \text{where } \operatorname{cov}(\rho_t, \varepsilon_t) = 0.$$

For other specifications of the stochastic unit root models see e.g. Granger and Swanson (1997); Leybourne et al. (2000); McCabe and Tremayne (1995); Sollis et al. (2000). We consider a simple case of (1) where $d_t = 0$ and $\nu_t = \varepsilon_t$, which gives:

$$y_t = (1 + b\varepsilon_{t-1}) \, y_{t-1} + \varepsilon_t. \tag{5}$$

For $b = 0$, (5) becomes a simple *URD* process, without any deterministic part (a random walk). The process (5) is nonstationary, since $1 + b^2\sigma^2 \geq 1$ (see Granger and Andersen, 1978). In Charemza et al. (2005) it is shown that, under the null hypothesis that $b = 0$ the Student t statistic for $\hat{b}$ in the regression equation:

$$\Delta y_t = \hat{b} y_{t-1} \Delta_{t-1} + e_t \tag{6}$$

where e_t are the regression residuals, has an asymptotic standard normal distribution. It is also shown that the similar test statistics can be formulated for a regression containing an intercept and for demeaned series of first differences of y_t. If the relationship between ν_t and ε_t is more complex, that is, where ν_t is described by a fully defined moving average process, it is possible to add augmentations to (6), as in the Augmented Dickey-Fuller test.

2   The Test

In order to decide whether a *URD* (possibly nonlinear, with a deterministic break) or a *URB* (without a deterministic break) model describes the series in a more accurate way,

a simple encompassing exercise has been performed. It is based on the following regression:

$$\Delta y_t = \hat{c} + \hat{b} y_{t-1} \Delta y_{t-1} + \hat{\gamma} B_t + \sum_{i=1}^{k} c_i \Delta y_{t-i} + e_t, \tag{7}$$

so that $\hat{c}$, $\hat{b}$ and $\hat{\gamma}$ are the estimates for c, b and γ. There are three sets of the hypotheses of interest to be considered:

(A) H_0: $b = 0$ and $\gamma = 0$ and H_1: $b \neq 0$ and/or $\gamma \neq 0$
(B) H_0: $\gamma = 0$ and H_1: $\gamma \neq 0$
(C) H_0: $b = 0$ and H_1: $b \neq 0$

The rejection of H_0 in (A) confirms that the unit root process is either subject to bilinearity, or a structural break. Further on, the rejection of H_0 in (B) confirms the existence of the deterministic structural break without much evidence of the *URB* process. If, however, H_0 is rejected in (A) and (C) and is not rejected in (B), this would implicitly confirm the existence of the *URB* and not the *URD* with break process.

For testing these sets of hypotheses we use Student t-statistics and, additionally in (A), the F statistic for testing the joint significance of b and γ on the basis of their estimators $\hat{b}$ and $\hat{\gamma}$. In (A), under the null hypothesis, the asymptotic distribution of the t-statistic for $\hat{\gamma}$, denoted as $t(\hat{\gamma})$ is standard normal. However, since the finite sample distribution of $t(\hat{b})$ is non-standard, the distribution of the F-statistic is unknown and has to be approximated numerically. The limit distribution of F-statistic in case of 'step' is given by the following theorem:

Theorem. *Let the series y_t be generated by*

$$y_t = y_{t-1} + \varepsilon_t, \varepsilon_t \sim IID(0, \sigma^2), y_0 = 0, t = 1, 2, \ldots, T. \tag{8}$$

Consider the regression model (7) *with B_t defined by* (4). *Denote by λ the ratio of time of break T_B to the sample size T and assume that it is constant (as T goes to infinity), that is*

$$\frac{T_B}{T} = \lambda = cons. \tag{9}$$

Under the null hypothesis of $b = \gamma = 0$, as $T \to \infty$ the F-statistic has a limit distribution of the form:

$$F \Rightarrow \tfrac{1}{2} \left[\frac{\left(\int_0^1 W_1(t) dW_2(t) \right)^2}{\int_0^1 W_1^2(t) dt} + \frac{(1-\sqrt{\lambda})(4-\lambda)}{\lambda(1+\sqrt{\lambda})} W_1^2(1) \right] \sim$$

$$\tfrac{1}{2} \left[\chi_1^2(1) + \frac{(1-\sqrt{\lambda})(4-\lambda)}{\lambda(1+\sqrt{\lambda})} \chi_2^2(1) \right], \tag{10}$$

where $\Rightarrow$ denotes weak convergence when $T \to \infty$, W_1, W_2 are two independent Wiener processes on [0,1] , *and $\chi_1^2(1), \chi_2^2(1)$, are two independent chi-squared distributions with one degree of freedom*[3] .

Proof of the Theorem is based on the following Lemma:

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[3]We are grateful to Mikhail Lifshits for additional comment on the independence of the chi squared variables in this theorem.

Lemma. *Let the series y_t be generated by* (8). *For the regression model* (7) *with* $BREAK_t$ *defined by* (4) *under the null hypothesis of* $b = \gamma = 0$, *as* $T \to \infty$:

1) $\sqrt{T}\hat{c} \Rightarrow \frac{\sigma}{\sqrt{\lambda}}W_1(1)$,

2) $T\hat{b} \Rightarrow \dfrac{\int_0^1 W_1(t)dW_2(t)}{\sigma \int_0^1 W_1^2(t)}$,

3) $\sqrt{T}\hat{\gamma} \Rightarrow \frac{\sigma}{\sqrt{\lambda}(1+\sqrt{\lambda})}W_1(1)$,

where $\Rightarrow$ *denotes weak convergence,* W_1, W_2 *are two independent Wiener processes on* [0,1] *and* λ *is defined by* (9).

The proofs of Lemma and the Theorem above are given in Appendix A.

It can also be shown that in (B), under the null hypothesis, the asymptotic distribution of $t(\hat{\gamma})$ is not normal and it depends on b. Moreover for $t(\hat{b})$ in (C), under the null hypothesis, that is for the Student-t statistic for the parameter b, the asymptotic normality is no longer the case, although the limit distribution remains well-defined as long as $0 < b \leq 1/\sqrt{T}$ (see Lifshits, 2002).

In order to facilitate testing, critical values for particular tests (assuming additionally the normality of ε_t) have been computed in a series of Monte Carlo experiments. Applying (7) to data generated by (5) with $b = 0$ gives approximate distributions of relevant statistics under the null hypothesis in (A). Similarly, applying (7) to data generated by (5) with non-zero values of b gives distributions of the statistics in (B) under the null hypothesis. In our experiments the parameter b has been chosen in such a way that the largest one corresponds to $b = 1/\sqrt{T}$ and the subsequent b's are proportionally smaller; b is defined as $b = d/\sqrt{T}$, where $d = 1$, 0.5, 0.25 and 0. For $d = 0$, the data generating process correspond to the null hypothesis in (A), otherwise it is that of (B). Value of T_B, that is, time indicator of a break, see (3) and/or (4), has been set at 0.85 T. This corresponds to the empirical example used in this paper (see further in Section 3). For each set of parameters and each sample size ($T = 1,000$, 1,500, 2,500), 100,000 replications were made. It turned out that sample size do not affect the results markedly.

Table 1 presents the relevant percentiles obtained in each experiment. The way of using this table is the following. To test the set of (A) hypotheses use values given in column headed $d = 0$. For testing the set of (B) hypotheses, it is necessary to compute $\delta = \hat{b}\sqrt{T}$. Next, choose a column headed by d to which δ could be rounded and use its percentiles as the critical values.

3   Empirical Results

In the empirical analysis we use daily (session-to session) data on prices and returns on 66 stock markets worldwide for the period from the 23rd of March 1992 until the 6th of December 2001. The length of the series varies from 1100 to 2534. It is assumed here that the possible breakpoint in the series, that is T_B in (3) and/or (4), is unknown. As the criteria for selection of the break date we use Zivot and Andrews (1992) criterion, according to which the break is in the period where the linear unit root t-statistic is the smallest. Other criteria considered by Vogelsang and Perron (1998) were also used, giving

Table 1: Critical values for the encompassing test

Test	percent.	$d = 0$	$d = 0.25$	$d = 0.50$	$d = 1$
	1%	-3.45	-3.04	-2.85	-2.56
	5%	-2.75	-2.38	-2.12	-1.61
$t(b)$	10%	-2.40	-1.98	-1.69	-1.05
	90%	2.40	3.11	3.96	5.96
	95%	2.75	3.75	5.41	9.09
	99%	3.45	5.62	10.37	16.19
	1%	as in N(0,1)	-2.95	-3.38	-3.87
	5%	as in N(0,1)	-2.04	-2.11	-2.93
$t(\gamma)$	10%	as in N(0.1)	-1.55	-1.60	-2.25
	90%	as in N(0,1)	1.83	2.19	2.76
	95%	as in N(0,1)	2.40	2.82	3.57
	99%	as in N(0,1)	3.54	4.04	4.96
	90%	4.47	N/A	N/A	N/A
$F(b,\gamma)$	95%	5.42	N/A	N/A	N/A
	99%	7.53	N/A	N/A	N/A

similar results.

It can be concluded that the *URB* process without a break encompasses the *URD* process with a break if for (7) the *F*-statistic is significant, b is significant under (A) and (B) and γ is not significant under (B). Analogously, the *URD* process with a deterministic break encompasses the *URB* process without a break if, in addition to the *F*-statistic being significant in (A), γ is significant under (B) and b is not significant under (C).

The detailed results of the encompassing exercise are presented at Table 2 at the Appendix B. They indicate that the *URB*-no break test shows, on the balance, some advantage over the *URD*-break approach. In 26 cases out of 66 the result is such that the t-statistic for $\hat{b}$ under the set of hypotheses (C) remains significant while the corresponding statistic for $\hat{\gamma}$ loses its significance under the set of hypotheses (B). Since in all these cases the *URD* model, while tested separately, appears to be significant, it might be concluded that a true bilinear unit root process might frequently be mistaken for the deterministic unit root process with a structural break. In 6 cases, however, the *URD* model seems to encompass the *URB* model. This includes the important USA market, for which the deterministic unit root model with a structural break appears to be superior. Among 6 countries for which the *URD* model encompasses *URB*, there is only one, Jordania, which is not within the *G25* group of the 25 richest (in terms of per capita real *GDP*) countries. Additionally we have found that only 6 countries belonging to the *G25* group are among the 26 for which the *URB* model encompasses *URD*. Hence, it seems that unit root bilinearity is more typical for the emerging and developing markets rather than for the mature markets.

Under (B), a relatively large number of insignificant results suggest that the problem of identification of underlying stochastic patterns for financial time series is far from being solved. True dynamics of these series is presumably more complicated and involves a mixture of a stochastic unit root process with that of a deterministic break. This, however,

requires further investigation.

4 Conclusions

Stochastic unit root modelling and, in particular, the bilinear unit root approach presented here, offers an attractive alternative to the traditional (deterministic) unit root analysis. The concept of the bilinear unit roots can substantially enrich the analysis traditionally conducted within the deterministic unit root framework. More specifically, a possible speculative bubble interpretation of the *URB* model and the computationally simple nature of its tests create an interesting tool for the analysis of ups and downs on financial markets. So far, speculative bubbles have been regarded as difficult to treat on the grounds of unit root econometrics (see e.g. Evans, 1991). Results presented here also reveal that a substantial number of the empirical financial time series exhibits unit root bilinearity. Testing for this particular form of the stochastic unit root is feasible and can be done without the need for developing specialised software. Finally the *URB* process, being lowly parameterised, does not require specific assumptions or additional tests regarding the nature or timing of the structural breaks.

Appendix A. Proofs of Lemma and Theorem

Proof of the Lemma. Consider the data generating process (*DGP*) given by equation:

$$y_t = y_{t-1} + \varepsilon_t, \varepsilon_t \sim IID(0, \sigma^2), y_0 = 0, t = 1, 2, \ldots, T. \tag{11}$$

For the parameters of the equation of interest (7) the usual *OLS* estimator is given as:

$$\left[\hat{c},\ \hat{b},\ \hat{\gamma}\right]' = (X'X)^{-1} X'Y, \tag{12}$$

where:

$$X = \begin{bmatrix} 1 & y_1 \Delta y_1 & 0 \\ 1 & y_1 \Delta y_1 & 0 \\ \ldots & \ldots & \ldots \\ 1 & y_{T_B-2} \Delta y_{T_B-2} & 0 \\ 1 & y_{T_B-1} \Delta y_{T_B-1} & 1 \\ \ldots & \ldots & \ldots \\ 1 & y_{T-1} \Delta y_{T-1} & 1 \end{bmatrix}, \text{ and } Y = \begin{bmatrix} \Delta y_2 \\ \Delta y_3 \\ \ldots \\ \Delta y_T \end{bmatrix}, \tag{13}$$

and T_B is a moment of a possible break. To obtain the asymptotic of matrices $X'X$ and $X'Y$ (with X and Y defined by (13)) under the *DGP* of (11) and the null hypothesis of $b = \gamma = 0$, let us apply the Donsker 's theorem and some results of Charemza et al. (2005), namely of the Statement, section 1 and Lemma C, sections (3) and (4), which

give:

$$T^{-1}\left(XX'\right)_{11} \Rightarrow 1, \ T^{-1}\left(XX'\right)_{12} \Rightarrow \sigma^2 \left(\int_0^1 W_1\left(t\right) dW_1\left(t\right) + 1\right),$$

$$T^{-1}\left(XX'\right)_{13} \Rightarrow 1 - \lambda, \ T^{-2}\left(XX'\right)_{22} \Rightarrow \sigma^4 \int_0^1 W_1^2\left(t\right) dt, \tag{14}$$

$$T^{-1}\left(XX'\right)_{23} \Rightarrow \left(1-\lambda\right)\sigma^2 \left(\int_0^1 W_1\left(t\right) dW_1\left(t\right) + 1\right), \ T^{-1}\left(XX'\right)_{31} \Rightarrow 1 - \lambda,$$

and

$$T^{-1/2}\left(X'Y\right)_{11} \Rightarrow \sigma W\left(1\right), \ T^{-1}\left(X'Y\right)_{21} \Rightarrow \sigma^3 \int_0^1 W_1\left(t\right) dW_2\left(t\right),$$

$$T^{-1/2}\left(X'Y\right)_{31} \Rightarrow \left(1 - \sqrt{\lambda}\right)\sigma W\left(1\right), \tag{15}$$

where $\left(XX'\right)_{ij}$ and $\left(X'Y\right)_{i1}$ $(i,j = 1,2,3)$ are corresponding elements of matrices $X'X$ and $X'Y$, $\Rightarrow$ denotes weak convergence, W_1, W_2 are two independent Wiener processes on [0,1] and λ is defined by (9).

Combination of (12), (14) and (15) after some usual algebra complete the proof of the Lemma.

Proof of the Theorem. The test statistics for $b = \gamma = 0$ in (7) has a form of the F-test:

$$F = \frac{\left(RSS_{(R)} - RSS_{(UR)}\right)/2}{RSS_{(UR)}/\left(T-4\right)}, \tag{16}$$

where $RSS_{(R)}$ is sum of squared *OLS*-residuals from the restricted regression, that is:

$$\Delta y_t = c + w_t, \quad \left(t = 2, 3, ..., T\right), \tag{17}$$

and $RSS_{(UR)}$ is sum of squared *OLS*-residuals from the unrestricted regression, that is from (7). For the restricted model (17) sum of squared *OLS*-residuals are:

$$RSS_{(R)} = \sum_{t=2}^T \hat{w}_t^2 = \sum_{t=2}^T \left(\Delta y_t\right)^2 - T\left(\frac{1}{T}\sum_{t=2}^T \Delta y_t\right)^2, \tag{18}$$

and, under the *DGP* of (11) and the null hypothesis of $b = \gamma = 0$, we get:

$$T\sum_{t=2}^T \left(\Delta y_t\right)^2 = T\sum_{t=2}^T \varepsilon_t^2 \Rightarrow \sigma^2 \tag{19}$$

and:

$$T\left(\frac{1}{T}\sum_{t=2}^T \Delta y_t\right)^2 = \left(\frac{1}{\sqrt{T}}\sum_{t=2}^T \varepsilon_t\right)^2 \Rightarrow \left[\sigma W_1\left(1\right)\right]^2. \tag{20}$$

Sum of squared *OLS*-residuals from the unrestricted regression of (7) with the use of (12) and (13) may be decomposed as:

$$RSS_{(UR)} = \sum_{t=2}^T e_t^2 = \sum_{t=2}^T \left(\Delta y_t\right)^2 + T\hat{c} + \hat{b}^2 \sum_{t=2}^T \left(y_{t-1}\Delta y_{t-1}\right)^2 - 2\hat{c}\sum_{t=2}^T \Delta y_{t-1}$$

$$-2\hat{b}\sum_{t=2}^T y_{t-1}\Delta y_{t-1}\Delta y_t + 2\hat{b}\hat{c}\sum_{t=2}^T y_{t-1}\Delta y_{t-1} - 2\hat{\gamma}\sum_{t=T_B}^T \Delta y_{t-1} \tag{21}$$

$$+2\hat{\gamma}\hat{c}\left(T - T_B\right) + 2\hat{\gamma}\hat{b}\sum_{t=T_B}^T y_{t-1}\Delta y_{t-1} + \hat{\gamma}^2\left(T - T_B\right).$$

From, (18), (19) and (20) we immediately obtain that, under the null hypothesis and as $T \to \infty$, the denominator in (16) will converge to σ^2, that is:

$$(T-4)^{-1} RSS_{(UR)} = (T-4)^{-1} \left(\sum_{t=2}^{T} (\varepsilon_t)^2 - T \left(\frac{1}{T} \sum_{t=2}^{T} \varepsilon_t \right)^2 \right) \Rightarrow \sigma^2. \qquad (22)$$

For the nominator in (16), applying the Lemma for decomposition (21), we obtain:

$$RSS_{(R)} - RSS_{(UR)} \Rightarrow \sigma^2 \frac{\left(\int_0^1 W_1(t)\, dW_2(t) \right)^2}{\int_0^1 W_1^2(t)\, dt} + \sigma^2 \frac{\left(1 - \sqrt{\lambda}\right)(4 - \lambda)}{\lambda \left(1 + \sqrt{\lambda}\right)} W_1^2(1). \qquad (23)$$

Combination of (16), (22) and (23) complete the proof the of Theorem.

Appendix B. Empirical Results

Table 2 presents the empirical results of the encompassing test exercise. After names of particular countries, the F-statistics for testing the joint significance of the parameters b and γ under the set of hypotheses (A) in (7) are given. Their significance is indicated in the third column as follows: '0' means no significance, '+', '++' and '+++' means that the statistic is significant respectively at 0.10, 0.05 and 0.01 level of significance. Column 4 gives the estimates of the t-statistics for b. The next two columns show their significance in (A) and (B). The last three columns show the values of Student-t statistics for γ in (A) and (B) respectively. The cases where the t-statistic for b under the set of hypotheses (B) remains significant while the corresponding statistic for γ loses its significance are in italics. The codes for countries which belong to the richest (in terms of per capita real *GDP,* the so called *G25* group) are marked by *.The 6 cases, for which the *URD* model seems to encompass the *URB* model are marked by boldfacing.

References

W.W. Charemza, M. Lifshits, and S. Makarova. Conditional testing for unit-root bilinearity in financial time series: some theoretical and empirical results. *Journal of Economic Dynamics and Control*, 29:63–96, 2005.

G.W. Evans. Pitfalls in testing for explosive bubbles in asset pricing. *American Economic Review*, 81:922–930, 1991.

C.W.J. Granger and A.P. Andersen. *An introduction to bilinear time series models*. Vandenhoeck and Ruprecht, Göttingen, 1978.

C.W.J. Granger and N.R. Swanson. An introduction to stochastic unit-root processes. *Journal of Econometrics*, 80:35–62, 1997.

D.I. Harvey and T.C. Mills. Tests for stationarity in series with endogenously determined structural change. In *European Meeting of the Econometric Society*, Venice, 2002.

T-H. Kim, S.J. Leybourne, and P. Newbold. Spurious rejection by perron test in the presence of a break. *Oxford Bulletin of Economics and Statistics*, 62:433–444, 2000.

S.J. Leybourne, B.P.M. McCabe, and A.R. Tremayne. Can economic time series be differenced to stationarity? *Journal of Business and Economic Statistics*, 14:435–446, 2000.

M. Lifshits. *Behaviour of statistical aggregates under small bilinearity*. St. Petersburg State University, mimeo, 2002.

B.P.M. McCabe and A.R. Tremayne. Testing a time series for difference stationarity. *The Annals of Statistics*, 23:1015–1028, 1995.

S. Ng and P. Perron. Lag length selection and the construction of unit root tests with good size and power. *Econometrica*, 69:1519–1554, 2001.

P. Perron. The great crash, the oil price shock and the unit root hypothesis. *Econometrica*, 57:1361–1401, 1989.

P. Perron. Testing for a unit root in a time series with a changing mean. *Journal of Business and Economic Statistics*, 8:153–162, 1990.

P. Perron. Further evidence on breaking trend functions in macroeconomic variables. *Journal of Econometrics*, 80:355–385, 1997.

R. Sollis, S.J. Leybourne, and P. Newbold. Stochastic unit roots modelling of stock price indices. *Applied Financial Economics*, 10:311–315, 2000.

T.J. Vogelsang and P. Perron. Additional tests for a unit root allowing for a break in the trend function at an unknown time. *International Economic Review*, 39:1073–1100, 1998.

E. Zivot and D.W.K. Andrews. Further evidence of the great crash, the oil shock and the unit root hypothesis. *Journal of Business and Economic Statistics*, 10:251–270, 1992.

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Table 2: Results of encompassing test

Code	Bilinearity stat.			Break statistic				
	$F(b,\gamma)$	**Snfce**	$t(b)$	**Snfce**		$t(\gamma)$	**Snfce**	
		(A)		**(A)**	**(B)**		**(A)**	**(B)**
ARGENTINA	33.59	+++	5.62	+++	0	−1.43	0	0
AUSTRALIA*	7.67	+++	−2.72	+++	+	−0.54	0	0
AUSTRIA*	5.45	++	−2.32	+++	0	−0.23	0	0
BANGLADESH	15.57	+++	3.89	+++	0	0.65	0	0
BELGIUM*	102.76	+++	10.01	+++	++	−1.63	0	0
BRAZIL	21.36	+++	4.60	+++	0	−0.40	+	0
CANADA*	8.44	+++	−2.60	+++	+	−1.30	0	0
CHILE	155.72	+++	12.48	+++	++	−0.13	0	0
CHINA	2.58	0	1.23	+++	0	−1.04	0	0
COLUMBIA	6.39	++	−2.53	+++	+	0.11	0	0
CROATIA	4.33	0	2.04	+++	0	0.42	0	0
CYPRUS*	34.76	+++	5.83	+++	0	−0.85	0	0
CZECH REPUBLIC	43.18	+++	6.57	+++	+	−0.06	0	0
DENMARK*	8.83	+++	−2.21	+++	0	−1.99	++	++
EGYPT	10.71	+++	2.87	++	0	−1.57	0	0
ESTONIA	1.33	0	−1.13	+++	0	0.23	0	0
FINLAND*	13.88	+++	−3.46	+++	+++	−1.38	+	0
FRANCE*	10.34	+++	2.67	+	0	−1.79	0	++
GERMANY*	10.19	+++	2.82	++	0	−1.50	0	0
GREECE	59.42	+++	7.62	+++	+	−1.17	0	0
HONG KONG*	6.96	++	2.48	+	0	−0.91	0	0
HUNGARY	18.39	+++	−4.25	+++	+++	−0.58	0	0
ICELAND*	13.85	+++	3.58	+++	0	−1.01	0	0
INDIA	44.84	+++	6.68	+++	+	−0.43	0	0
INDONESIA	46.91	+++	6.79	+++	+	−0.90	0	0
IRELAND*	7.48	++	−2.62	+++	+	−0.79	0	0
ISRAEL*	3.73	0	1.19	+++	0	−1.52	0	0
ITALY*	5.58	++	−1.76	+++	0	−1.57	+	0
JAPAN*	5.50	++	2.09	+++	0	−1.06	0	0
JORDANIA	15.81	+++	3.47	+++	0	1.93	0	++
KOREA, REP OF	7.86	+++	2.76	++	0	0.49	0	0
LATVIA	17.39	+++	4.17	+++	0	0.18	0	0
LEBANON	46.57	+++	6.78	+++	+	−0.74	0	0
LITHUANIA	45.72	+++	6.76	+++	+	−0.17	0	0
LUXEMBOURG*	63.35	+++	7.42	+++	+	−2.87	+	+++
MALAYSIA	21.76	+++	4.66	+++	0	−0.22	0	0
MAURITIUS	11.40	+++	3.05	++	0	−1.44	0	0
MEXICO	72.29	+++	8.48	+++	+	−0.59	0	0
MOROCCO	68.73	+++	8.24	+++	+	−0.90	0	0
NETHERLANDS*	10.18	+++	2.01	+++	0	−2.48	+	+++
NIGERIA	0.27	0	−0.52	+++	0	0.07	0	0
NORWAY*	5.91	++	−2.00	+++	0	−1.37	+	0

Table 2 (cont.): Results of encompassing test

Code	Bilinearity stat.			Break statistic				
	$F(b,\gamma)$	Snfce	$t(b)$	Snfce		$t(\gamma)$	Snfce	
		(A)		(A)	(B)		(A)	(B)
NEW ZEALAND*	0.03	0	0.17	+++	0	-0.02	0	0
PAKISTAN	18.89	+++	4.34	+++	0	-0.27	0	0
PERU	85.86	+++	9.26	+++	++	-0.45	0	0
PHILIPPINES	90.71	+++	9.48	+++	++	-0.88	0	0
POLAND	5.33	+	-2.30	+++	0	-0.20	0	0
PORTUGAL	9.12	+++	-2.61	+++	+	-1.51	+	0
ROMANIA	2.65	0	-1.50	+++	0	0.64	0	0
RUSSIA	3.32	0	-1.82	+++	0	0.01	0	0
SOUTH AFRICA	49.80	+++	7.06	+++	+	0.00	0	0
SINGAPORE*	22.89	+++	4.76	+++	0	-0.44	0	0
SLOVAKIA	9.14	+++	-2.83	+++	++	1.08	0	0
SLOVENIA	234.24	+++	15.29	+++	++	0.71	0	0
SPAIN*	6.06	++	-2.26	+++	0	-0.97	0	0
SRI LANKA	273.56	+++	16.50	+++	+++	1.14	0	0
SWITZERLAND*	16.14	+++	3.45	+++	0	-2.05	0	++
SWEDEN*	18.65	+++	-4.15	+++	+++	-1.18	0	0
TAIWAN, CHINA	0.75	0	0.81	+++	0	-0.32	0	0
THAILAND	39.07	+++	6.25	+++	+	0.00	0	0
TURKEY	20.93	+++	-4.51	+++	+++	-0.74	0	0
U. K.*	18.65	+++	4.02	+++	0	-1.58	0	0
UNITED STATES*	5.46	++	1.43	+++	0	-1.85	+	++
VENEZUELA	10.84	+++	3.14	++	0	-1.00	0	0
ZIMBABWE	283.14	+++	16.82	+++	+++	0.60	0	0